Solutions:

1. If $x_n = \frac{n}{5^n} = \frac{n}{(1+4)^n} = \frac{n}{\binom{n}{0}+\binom{n}{1}4+\binom{n}{2}4^2+\cdots+\binom{n}{n}4^n}$, then $\lim_{n\to\infty} x_n = 0$. Because, if we choose the natural number $N > \frac{1+8\varepsilon}{8\varepsilon}$, then for all $n \geq N$ we have $x_n = \frac{n}{5^n} = \frac{n}{(1+4)^n} = \frac{n}{\binom{n}{0}+\binom{n}{1}4+\binom{n}{2}4^2+\cdots+\binom{n}{n}4^n} < \frac{n}{\binom{n}{2}4^2} = \frac{n}{8n(n-1)} = \frac{1}{8(n-1)} < \varepsilon$.

2. Let $0 < \underline{lim} y_n < +\infty$. Then for every $\varepsilon > 0$, there exists a natural number N_1 such that $N \geq N_1$ implies $|inf\{y_n : n \geq N\} - y| < \varepsilon$. Thus $inf\{y_n : n \geq N\} > y - \varepsilon$ for $N \geq N_1$, which means that $inf\{y_n : n \geq N_1\} > y - \varepsilon$. So we get $y_n > y - \varepsilon \ \ for\ all\ n \geq N_1$. If we take $\varepsilon = y - m$, where $0 < m < y$, then there exists a natural number N_1 such that $n \geq N_1$ implies $m = y - \varepsilon < y_n$.

On the other hand, since $\lim_{n\to\infty} x_n = +\infty$ then for every real number M there is a natural number N_2 such that $n \geq N_2$ implies $x_n > \frac{M}{m}$. If we take $N = max\{N_1, N_2\}$, then $n \geq N$ implies $x_n y_n > M$. So for every real number M there is a natural number N such that $n \geq N$ implies $inf\{x_n y_n : n \geq N\} > M$. Hence we get $\underline{lim}(x_n y_n) = +\infty$.

3. a) $\lim_{x\to 0} \frac{\sqrt[n]{1+x}-1}{x} = \lim_{x\to 0} \frac{\left(\sqrt[n]{1+x}-1\right)\left(\sqrt[n]{(1+x)^{n-1}}+\sqrt[n]{(1+x)^{n-2}}+\cdots+\sqrt[n]{1+x}+1\right)}{x\left(\sqrt[n]{(1+x)^{n-1}}+\sqrt[n]{(1+x)^{n-2}}+\cdots+\sqrt[n]{1+x}+1\right)} = \lim_{x\to 0} \frac{x}{x\left(\sqrt[n]{(1+x)^{n-1}}+\sqrt[n]{(1+x)^{n-2}}+\cdots+\sqrt[n]{1+x}+1\right)}$

$= \lim_{x\to 0} \frac{1}{\sqrt[n]{(1+x)^{n-1}}+\sqrt[n]{(1+x)^{n-2}}+\cdots+\sqrt[n]{1+x}+1} = \frac{1}{n}$.

b) $\lim_{x\to+\infty}\left(\frac{x^3}{3x^2-4} - \frac{x^2}{3x+2}\right) = \lim_{x\to+\infty}\left(\frac{3x^4+2x^3-3x^4+4x^2}{(3x^2-4)(3x+2)}\right) = \lim_{x\to+\infty}\left(\frac{2x^3+4x^2}{9x^3+6x^2-12x+8}\right) = \frac{2}{9}$.

4. For the right-hand limit at $x = 1$ we have,

$$\lim_{x\to 1^+} f(x) = \lim_{\substack{h\to 0\\ h>0}} f(1+h) = \lim_{\substack{h\to 0\\ h>0}} \left(\frac{h}{3+2^{1/(-h)}}\right) = 0.$$

For the left-hand limit at $x = 1$ we have

$$\lim_{x\to 1^-} f(x) = \lim_{\substack{h\to 0\\ h>0}} f(1-h) = \lim_{\substack{h\to 0\\ h>0}} \left(\frac{-h}{3+2^{1/h}}\right) = 0.$$

Since $\lim_{x\to 1^+} f(x) = 0 = \lim_{x\to 1^-} f(x)$, then the function is continuous at $x = 1$.

5. If we write $|f(x) - f(y)| = \left|\frac{x-1}{3x-2} - \frac{y-1}{3y-2}\right| = \left|\frac{3xy-2x-3y+2-3xy+3x+2y-2}{(3x-2)(3y-2)}\right| = \left|\frac{x-y}{(3x-2)(3y-2)}\right|$, then for all x and y in $E = [2, +\infty)$ we have

$$|f(x) - f(y)| = \left|\frac{x-y}{(3x-2)(3y-2)}\right| < \frac{|x-y|}{16}.$$

If we write $|f(x) - f(y)| = \left|\frac{x-y}{(3x-2)(3y-2)}\right| < \frac{|x-y|}{16} < \varepsilon$ then we can choose $\delta > 0$ such that $|x-y| < 16\varepsilon = \delta$.