

$S^{(1)} = 1/2 ; m_{S1} = 1/2, -1/2$
 $S^{(2)} = 1/2 ; m_{S2} = 1/2, -1/2$
 $S = S^{(1)} + S^{(2)} \rightarrow S = |S_1 - S_2| \dots (S_1 + S_2) \rightarrow S = 1, 0$

Für alle anderen Zustände muss man die Eigenwerte berechnen:
 $|1/2, 1/2; 1/2, 1/2\rangle = |\uparrow\uparrow\rangle \quad m=1$
 $|1/2, 1/2; 1/2, -1/2\rangle = |\uparrow\downarrow\rangle \quad m=0$
 $|1/2, -1/2; 1/2, 1/2\rangle = |\downarrow\uparrow\rangle \quad m=0$
 $|1/2, -1/2; 1/2, -1/2\rangle = |\downarrow\downarrow\rangle \quad m=-1$

$S^{(1)} = 1/2 ; m_{S1} = 1/2, -1/2$
 $S^{(2)} = 1/2 ; m_{S2} = 1/2, -1/2$
 $S = S^{(1)} + S^{(2)} \rightarrow S = |S_1 - S_2| \dots (S_1 + S_2) \rightarrow S = 1, 0$

Für alle anderen Zustände muss man herausfinden:

Elke edilen dört tane kuma bunların her birini St'ın ödemesinden;

$$\begin{aligned}
 \bullet S_z | \uparrow_{1/2}, \uparrow_{1/2} \rangle | \uparrow_{1/2}, \downarrow_{1/2} \rangle &= (S_z^{(1)} + S_z^{(2)}) | \uparrow_{1/2}, \uparrow_{1/2} \rangle | \uparrow_{1/2}, \downarrow_{1/2} \rangle \\
 S_{1 \text{ ms}} \quad S_{2 \text{ ms}} &= \underbrace{\left(S_z^{(1)} | \uparrow_{1/2}, \uparrow_{1/2} \rangle \right)}_{\frac{1}{2} \hbar | \uparrow_{1/2}, \uparrow_{1/2} \rangle} | \uparrow_{1/2}, \downarrow_{1/2} \rangle + | \uparrow_{1/2}, \uparrow_{1/2} \rangle \underbrace{\left(S_z^{(2)} | \uparrow_{1/2}, \downarrow_{1/2} \rangle \right)}_{\frac{1}{2} \hbar | \uparrow_{1/2}, \downarrow_{1/2} \rangle} \\
 &= \frac{1}{2} \hbar | \uparrow_{1/2}, \uparrow_{1/2} \rangle | \uparrow_{1/2}, \downarrow_{1/2} \rangle + \frac{1}{2} \hbar | \uparrow_{1/2}, \uparrow_{1/2} \rangle | \uparrow_{1/2}, \downarrow_{1/2} \rangle \\
 &= \left(\frac{1}{2} \hbar + \frac{1}{2} \hbar \right) | \uparrow_{1/2}, \uparrow_{1/2} \rangle | \uparrow_{1/2}, \downarrow_{1/2} \rangle \\
 &= \hbar | \uparrow_{1/2}, \uparrow_{1/2} \rangle | \uparrow_{1/2}, \downarrow_{1/2} \rangle, \quad m = m_{S1} + m_{S2} = \frac{1}{2} \hbar
 \end{aligned}$$

$$\begin{aligned} \bullet S_2 |1/2, 1/2\rangle |1/2, -1/2\rangle &= (S_2^{(1)} + S_2^{(2)}) |\uparrow\rangle |\downarrow\rangle \\ \begin{matrix} |\uparrow\rangle \\ m_{s1} = 1/2 \end{matrix} & \quad \begin{matrix} |\downarrow\rangle \\ m_{s2} = -1/2 \end{matrix} &= \underbrace{(S_2^{(1)} |\uparrow\rangle)}_{\frac{1}{2} |\uparrow\rangle} |\downarrow\rangle + |\uparrow\rangle \underbrace{(S_2^{(2)} |\downarrow\rangle)}_{-\frac{1}{2} |\downarrow\rangle} \\ &= \left(\frac{1}{2} - \frac{1}{2} \right) |\uparrow\rangle |\downarrow\rangle \\ &= 0 \frac{1}{2} |\uparrow\rangle |\downarrow\rangle \quad m = m_{s1} + m_{s2} = 0 \end{aligned}$$

$$\begin{aligned} \bullet \quad S_z |1/2, -1/2\rangle |1/2, 1/2\rangle &= (S_z^{(1)} + S_z^{(2)}) |\downarrow\rangle |\uparrow\rangle \\ &= (S_z^{(1)} |\downarrow\rangle) |\uparrow\rangle + |\downarrow\rangle (S_z^{(2)} |\uparrow\rangle) \\ &= \left(-\frac{\hbar}{2} + \frac{\hbar}{2}\right) |\downarrow\rangle |\uparrow\rangle \\ &= 0 \frac{\hbar}{2} |\downarrow\rangle |\uparrow\rangle \quad m = m_1 + m_2 = 0 \end{aligned}$$

- $S_z |1/2, -1/2\rangle |1/2, -1/2\rangle = (S_z^{(1)} |1/2\rangle |1/2\rangle + |1/2\rangle (S_z^{(2)} |1/2\rangle)$
 $= (-\frac{1}{2} - \frac{1}{2}) |1/2\rangle |1/2\rangle$
 $= -1 |1/2\rangle |1/2\rangle$

Bu yarıdıkların ilk birkaçı doğru gözlenmiyor! Çünkü m 'in $+5$ 'den -5 'e tümseki adimlarla ilerlemesi gerekir. Böylece $S=1$ durumu elde edilir. Ancak $m=0$ olan "fonlarda" bir durum daha vardır. Bu sonucu kontrol için $|1\rangle|1\rangle$ durumuna S_- operatörüne uygulayalım;

$$S_- |S, m_S\rangle = \hbar \sqrt{S(S+1) - m_S(m_S \mp 1)} |S, m_S \mp 1\rangle$$

$$\begin{aligned}
 \bullet \quad S_- |1/2, 1/2\rangle |1/2, 1/2\rangle &= (S_-^{(1)} + S_-^{(2)}) |1/2, 1/2\rangle |1/2, 1/2\rangle \\
 &= \left[S_-^{(1)} |1/2, 1/2\rangle \right] |1/2, 1/2\rangle + |1/2, 1/2\rangle \left[S_-^{(2)} |1/2, 1/2\rangle \right] \\
 &\quad \begin{array}{c} \underbrace{|1/2, 1/2\rangle}_{|S, m_S\rangle = |1, 1\rangle} \\ \uparrow \quad \uparrow \\ S_T \quad m_{S, toplam} \end{array} \quad \begin{array}{c} \hbar \sqrt{\frac{1}{2}(\frac{1}{2}+1) - \frac{1}{2}(\frac{1}{2}-1)} |1/2, -1/2\rangle \\ \hbar |1/2, -1/2\rangle \end{array} \\
 &= \hbar |1/2, -1/2\rangle |1/2, 1/2\rangle + \hbar |1/2, 1/2\rangle |1/2, -1/2\rangle \\
 &= \hbar \left[|1/2, -1/2\rangle |1/2, 1/2\rangle + |1/2, 1/2\rangle |1/2, -1/2\rangle \right] \\
 S_- |1, 1\rangle &= \hbar \left[|1/2, 1/2\rangle |1/2, 1/2\rangle + |1/2, 1/2\rangle |1/2, 1/2\rangle \right]
 \end{aligned}$$

$$\begin{array}{l}
 |S, m_S\rangle \\
 |1, 1\rangle = |1/2, 1/2\rangle |1/2, 1/2\rangle \\
 |1, 0\rangle = \frac{1}{\sqrt{2}} \left[|1/2, 1/2\rangle |1/2, -1/2\rangle + |1/2, -1/2\rangle |1/2, 1/2\rangle \right] \\
 |1, -1\rangle = |1/2, -1/2\rangle |1/2, -1/2\rangle
 \end{array} \left. \vphantom{\begin{array}{l} |S, m_S\rangle \\ |1, 1\rangle \\ |1, 0\rangle \\ |1, -1\rangle \end{array}} \right\} \begin{array}{l} S=1 \text{ içi 3 durum} \\ (\text{triplet}) \end{array}$$

$S=1, m=0$ durumuna ortogonal durum;

$$|0, 0\rangle = \frac{1}{\sqrt{2}} \left[|1/2, 1/2\rangle |1/2, -1/2\rangle - |1/2, -1/2\rangle |1/2, 1/2\rangle \right] \quad \begin{array}{l} S=0 \text{ (tekli durum)} \\ (\text{singlet}) \end{array}$$

Kontrol için;

$$\begin{aligned}
 \bullet \quad S_- |1, 0\rangle &= S_- \left[\frac{1}{\sqrt{2}} |1/2, 1/2\rangle |1/2, -1/2\rangle + \frac{1}{\sqrt{2}} |1/2, -1/2\rangle |1/2, 1/2\rangle \right] \\
 &= \left[\frac{1}{\sqrt{2}} (S_-^{(1)} |1/2, 1/2\rangle) |1/2, -1/2\rangle + \frac{1}{\sqrt{2}} |1/2, 1/2\rangle (S_-^{(2)} |1/2, -1/2\rangle) \right] \\
 &\quad + \left[\frac{1}{\sqrt{2}} (S_-^{(1)} |1/2, -1/2\rangle) |1/2, 1/2\rangle + \frac{1}{\sqrt{2}} |1/2, -1/2\rangle (S_-^{(2)} |1/2, 1/2\rangle) \right]
 \end{aligned}$$

$$S_- |1,0\rangle = \frac{\hbar}{\sqrt{2}} |\downarrow\uparrow\downarrow\rangle + \frac{\hbar}{\sqrt{2}} |\downarrow\downarrow\uparrow\rangle$$

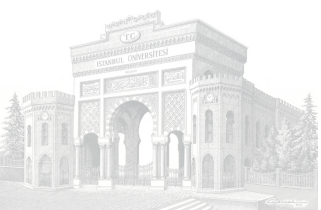
$$= \frac{2\hbar}{\sqrt{2}} |\downarrow\downarrow\uparrow\rangle$$

$$S_- |1,0\rangle = \hbar\sqrt{2} |1,-1\rangle, \quad S_+ |1,0\rangle \rightarrow |1,1\rangle$$

$$\begin{aligned} \bullet S_- |0,0\rangle &= (S_-^{(1)} + S_-^{(2)}) \left[\frac{1}{\sqrt{2}} |\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}} |\downarrow\uparrow\rangle \right] \\ &= \frac{1}{\sqrt{2}} \left(\underbrace{S_-^{(1)} |\uparrow\rangle}_{\hbar |\downarrow\rangle} \right) |\downarrow\rangle + \frac{1}{\sqrt{2}} |\uparrow\rangle \left(\cancel{S_-^{(2)} |\downarrow\rangle} \right) - \frac{1}{\sqrt{2}} \left(\cancel{S_-^{(1)} |\downarrow\rangle} \right) |\uparrow\rangle \\ &\quad - \frac{1}{\sqrt{2}} |\downarrow\rangle \left(\underbrace{S_-^{(2)} |\uparrow\rangle}_{\hbar |\downarrow\rangle} \right) \\ &= \frac{\hbar}{\sqrt{2}} |\downarrow\downarrow\rangle - \frac{\hbar}{\sqrt{2}} |\downarrow\downarrow\rangle \end{aligned}$$

$$S_- |0,0\rangle = 0$$

$$S_+ |0,0\rangle = 0$$



$$\left. \begin{array}{l} S^{(1)} = 1, \\ S^{(2)} = 2, \end{array} \right\} S^{\text{Top}} = \boxed{3}, 1$$

$$\downarrow$$
 sirket $S = 3$ ve $m_S = 1$ dur, $m_{S_4} = 1$ $S, m_S = \underline{\underline{13, 1}}$

$$m_{r1} = 0, m_{r2} = 1 \rightarrow m_r = 1 \checkmark$$

$$a = \sqrt{\frac{6}{15}}, \quad b = \sqrt{\frac{1}{15}}, \quad c = \sqrt{\frac{8}{15}}$$

$$|p(r_1, r_2)| = |\langle r_1, r_2 | \psi \rangle|^2 = \frac{1}{4\pi}$$

5.3. Spin: aşağı olan bir elektron Hidrojen atomunun χ_{510} durumundadır. Yukarıdaki elektronun (protonun spin hariç) toplam açısal momentumunun karesi ölçüldüğünde hangi değerler hangi olasılıklar ile elde edilir?

$$e^- \rightarrow s = 1/2$$

$$\text{spin aşağı} \rightarrow m_s = -1/2$$

$$\chi_{510} \rightarrow n=5 \\ l=1 \\ m_l=0$$

$$|L-S| \leq j \leq L+S$$

$$|1-5| \leq j \leq 1+5$$

$$|1-1/2| \leq j \leq 1+1/2$$

$$j: 3/2, 1/2$$

$$m_j = m_l + m_s = 0 - 1/2 = -1/2$$

$$\chi_{\text{tüm}} = \chi_{510} \chi_{\text{spin}} = |\chi_{510}\rangle |\frac{1}{2}, -\frac{1}{2}\rangle$$

$$|1,0\rangle |\frac{1}{2}, -\frac{1}{2}\rangle = a |\frac{3}{2}, -\frac{1}{2}\rangle + b |\frac{1}{2}, -\frac{1}{2}\rangle$$

$$|1,0\rangle |\frac{1}{2}, -\frac{1}{2}\rangle = \sqrt{\frac{2}{3}} |\frac{3}{2}, -\frac{1}{2}\rangle + \sqrt{\frac{1}{3}} |\frac{1}{2}, -\frac{1}{2}\rangle$$

1x1/2	3/2			
+1 +1/2	+3/2	3/2 1/2		
	1	+1/2 +1/2		
+1 -1/2	0 +1/2	1/3 2/3	3/2 1/2	
		2/3 -1/3	-1/2 -1/2	
		0 -1/2	2/3 1/3	3/2
		-1 +1/2	1/3 -2/3	-3/2
			-1 -1/2	1

Clebsch - Gordan katsayılarından;

$$a = \sqrt{\frac{2}{3}}, \quad b = \sqrt{\frac{1}{3}}$$

$$\hat{J}^2 |j, m_j\rangle = \hbar^2 j(j+1) |j, m_j\rangle$$

$$\hat{J}^2 |1,0\rangle |\frac{1}{2}, -\frac{1}{2}\rangle = \sqrt{\frac{2}{3}} \underbrace{\hat{J}^2 |\frac{3}{2}, -\frac{1}{2}\rangle}_{\hbar^2 \frac{15}{4}} + \sqrt{\frac{1}{3}} \underbrace{\hat{J}^2 |\frac{1}{2}, -\frac{1}{2}\rangle}_{\hbar^2 \frac{3}{4}}$$

$$P(j = \frac{3}{2}) = \left| \langle \frac{3}{2}, m_j | (|1,0\rangle |\frac{1}{2}, -\frac{1}{2}\rangle) \right|^2 = \frac{2}{3}$$

$$P(j = \frac{1}{2}) = \left| \langle \frac{1}{2}, m_j | (|1,0\rangle |\frac{1}{2}, -\frac{1}{2}\rangle) \right|^2 = \frac{1}{3}$$



5.4. Bir Hidrojen atomunda: elektronun toplam (tam) dalgac fonksiyonu,

$$\chi = R_{21} \left(\sqrt{\frac{1}{3}} Y_1^0 \chi_+ + \sqrt{\frac{2}{3}} Y_1^1 \chi_- \right)$$

olarak verilmiştir,

- a) L^2 operatörüne hangi değerler hangi olasılıklarla elde edilir?
- b) L_z " " " " " " "
- c) S^2 " " " " " " "
- d) S_z " " " " " " "
- e) J^2 " " " " " " "
- f) " " " " " " "

g) Parçacığın konumu ele alınarak, (r, θ, ϕ) uzayında bulunma olasılık yoğunluğu nedir?

$$\chi = R_{21} \left(\sqrt{\frac{1}{3}} Y_1^0 \chi_+ + \sqrt{\frac{2}{3}} Y_1^1 \chi_- \right)$$

$n=2$
 $l=1$

$l=1$
 $m=0$

$l=1$
 $m=0$

$$\chi_{\pm} = \left| \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}} \right\rangle$$

$$R_{nc} Y_c^m$$

$$E_n = -\frac{13.6 \text{ eV}}{n^2} \rightarrow E_2 = -\frac{13.6 \text{ eV}}{4}$$

a: $L^2 |n, l, m\rangle = \hbar^2 l(l+1) |n, l, m\rangle$

$$L^2 \chi = L^2 \left(R_{21} \sqrt{\frac{1}{3}} Y_1^0 \chi_+ + R_{21} \sqrt{\frac{2}{3}} Y_1^1 \chi_- \right)$$

$$= \hbar^2 1(1+1) \chi \rightarrow L^2 = 2\hbar^2, \text{ \%100 olasılıkla,}$$

$$P(L^2 = 2\hbar^2) = 1$$

b: $L_z |n, l, m\rangle = m\hbar |n, l, m\rangle$

$$L_z \chi = R_{21} \left[\sqrt{\frac{1}{3}} \underbrace{L_z Y_1^0}_{0\hbar Y_1^0} \chi_+ + \sqrt{\frac{2}{3}} \underbrace{L_z Y_1^1}_{\hbar Y_1^1} \chi_- \right]$$

$$L_z; 0\hbar, \hbar$$

$$P(L_z = 0\hbar) = \frac{1}{3}, \quad P(L_z = \hbar) = \frac{2}{3}$$

c) $\tilde{S} |S, m_S\rangle = \hbar S(S+1) |S, m_S\rangle$

$$\tilde{S} \chi = R_{11} \left(\sqrt{\frac{1}{3}} \underbrace{Y_1^0}_{\hbar \frac{1}{2}(\frac{3}{2})} \tilde{S} \chi_+ + \sqrt{\frac{2}{3}} \underbrace{Y_1^1}_{\hbar \frac{1}{2}(\frac{3}{2})} \tilde{S} \chi_- \right) \Rightarrow \tilde{S} = \frac{3\hbar^2}{4} \quad \%100$$

$P(\tilde{S}_z = \hbar/2) = 1 //$

d) $S_z |S, m_S\rangle = m_S \hbar |S, m_S\rangle$

$$S_z \chi = R_{11} \left[\sqrt{\frac{1}{3}} \underbrace{Y_1^0}_{\frac{1}{2} \hbar} S_z | \frac{1}{2}, +\frac{1}{2} \rangle + \sqrt{\frac{2}{3}} \underbrace{Y_1^1}_{-\frac{1}{2} \hbar} S_z | \frac{1}{2}, -\frac{1}{2} \rangle \right]$$

$$S_z \chi = \hbar/2, \quad P(S_z = \hbar/2) = 1/3$$

$$S_z \chi = -\hbar/2, \quad P(S_z = -\hbar/2) = 2/3$$

e) $l=1, m_l=0, 1$

$$S = 1/2, m_S = -1/2, +1/2$$

$$\chi = \underbrace{\sqrt{\frac{1}{3}} R_{11} Y_1^0 \chi_+}_{\substack{l=1, S=1/2 \\ m_l=0, m_S=1/2 \\ j=3/2, 1/2 \\ m_j=1/2}} + \underbrace{\sqrt{\frac{2}{3}} R_{11} Y_1^1 \chi_-}_{\substack{l=1, S=1/2 \\ m_l=1, m_S=-1/2 \\ j=3/2, 1/2 \\ m_j=1/2}}$$

$$a | \frac{3}{2}, \frac{1}{2} \rangle + b | \frac{1}{2}, \frac{1}{2} \rangle \quad c | \frac{1}{2}, \frac{1}{2} \rangle + d | \frac{3}{2}, \frac{1}{2} \rangle$$

$j \quad m_j \quad j \quad m_j$

$$\left. \begin{aligned} a &= c = \sqrt{\frac{2}{3}} \\ b &= d = \sqrt{\frac{1}{3}} \end{aligned} \right\} \text{ tablodan}$$

$$\chi = R_{11} \left(\sqrt{\frac{1}{3}} \left(\sqrt{\frac{2}{3}} | \frac{3}{2}, \frac{1}{2} \rangle + \sqrt{\frac{1}{3}} | \frac{1}{2}, \frac{1}{2} \rangle \right) + \sqrt{\frac{2}{3}} \left(\sqrt{\frac{1}{3}} | \frac{1}{2}, \frac{1}{2} \rangle + \sqrt{\frac{2}{3}} | \frac{3}{2}, \frac{1}{2} \rangle \right) \right)$$

$$\chi = R_{11} \left(\frac{2\sqrt{2}}{3} | \frac{3}{2}, \frac{1}{2} \rangle + \frac{1}{3} | \frac{1}{2}, \frac{1}{2} \rangle \right) \quad j = \frac{1\sqrt{15}}{4}, \quad P = \frac{8}{5}$$

$$j = \frac{3\hbar}{4}, \quad P = 1/3$$

9)

$$|\chi\rangle \sim \chi^* \chi$$

$$|\chi\rangle \sim R_{11}^* \left(\sqrt{\frac{1}{3}} y_1^0 x_+^* + \sqrt{\frac{2}{3}} y_1^1 x_-^* \right) R_{11} \left(\sqrt{\frac{1}{3}} y_1^0 x_+ + \sqrt{\frac{2}{3}} y_1^1 x_- \right)$$

χ^* χ

$$|\chi\rangle \sim |R_{11}\rangle \left[\frac{1}{3} |y_1^0\rangle + \frac{2}{3} |y_1^1\rangle \right] \quad y_1^0 y_1^1 = 0$$

$$R_{11} = \frac{1}{\sqrt{2\pi}} a^{-3/2} \frac{r}{a} e^{-r/a}$$

$$x_+ x_- = 0$$

$$y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta, \quad y_1^1 = \sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi}$$

$$|\chi\rangle \sim \frac{1}{\sqrt{4\pi}} \frac{r}{a^2} e^{-r/a} \left[\frac{1}{3} \frac{r}{4\pi} \cos\theta + \frac{2}{3} \frac{r}{8\pi} \sin\theta \right] = \frac{1}{\sqrt{4\pi}} \frac{r^2}{a^2} e^{-r/a} \frac{1}{4\pi}$$

$$|\chi\rangle \sim \frac{1}{36\pi a^2} r^2 e^{-r/a}$$

