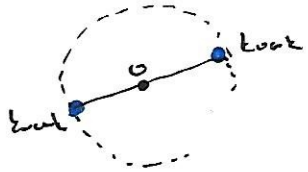
3.1. Aynı m kütlesine sahip ve $V(r) = kr$ (k pozitif bir sabit) potansiyeli ile etkileşen iki kuarkın bağlı durumlarını düşünün. Bohr modelini kullanarak, çembersel yörünge durumunda, sistemin hızını, yarıçapını ve enerjisini bulun. Sistemin n durumundan m durumuna geçmesi hâlinde yayınlanan ışımanın frekansını bulun.



Çembersel olarak hareket eden iki kuark, bir Hidrojen atomundaki bir proton ve bir elektrona benzerdir. Bunlar arasındaki kuvvet: $\vec{F} = -\vec{\nabla} V(r)$ (merkezcil kuvvet)

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}, \quad m_1 = m_2 = m$$

$\mu = \frac{m}{2}$ 'dır. (indirgenmiş kütle)

$$\vec{F}_{mer} + \vec{F} = \vec{0}$$

$$m\frac{V^2}{r}\hat{e}_s + \left(-\vec{\nabla} V(r)\right) = \vec{0}$$

$$m\frac{V^2}{r}\hat{e}_s + \left(-\frac{d}{dr}V(r)\,\hat{e}_s\right) = 0 \Rightarrow \frac{dV(r)}{dr} = m\frac{V^2}{r} \Rightarrow \frac{d}{dr}(kr) = m\frac{V^2}{r}$$

$$\Rightarrow \frac{mV^2}{r} = k \Rightarrow mV^2 = kr$$

Yörüngesel açısal momentumun Bohr kuantizasyonu:

$$mVr = n\hbar$$

$$r_n = \frac{n\hbar}{mV} \Rightarrow r_n = \frac{n\hbar}{\mu V}$$

↑ indirgenmiş kütle

$$\Rightarrow \mu V^2 = kr, \quad \mu: \text{indirgenmiş kütle}$$

$$\Rightarrow \mu V^2 = kr \Rightarrow \mu V_n^2 = k\frac{n\hbar}{\mu V}$$

$$\boxed{V_n = \left[\frac{\hbar k}{\mu^2}\right]^{1/3} n^{1/3}}$$

- $$r_n = \frac{n\hbar}{mV_n} = \frac{n\hbar}{\left[\frac{\hbar k}{\mu^2}\right]^{1/3}\mu n^{1/3}} \Rightarrow \boxed{r_n = \left[\frac{\hbar^2}{\mu k}\right]^{1/3} n^{2/3}}$$

- $$E_n = \frac{1}{2}\mu V_n^2 + k r_n = \frac{1}{2}\mu\left[\frac{\hbar k}{\mu^2}\right]^{2/3} n^{2/3} + k\left[\frac{\hbar^2}{\mu k}\right]^{1/3} n^{2/3}$$

$$\boxed{E_n = \frac{3}{2}\left(\frac{\hbar^2 \varepsilon^2}{m}\right)^{1/3}(n^{2/3})}$$

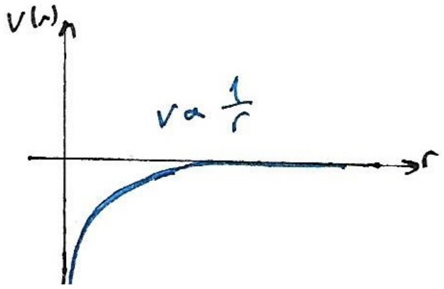
- $E_n - E_m = \hbar\omega_{nm} \Rightarrow \omega_{nm} = \dfrac{E_n - E_m}{\hbar} = \dfrac{3}{2\hbar}\left(\dfrac{\hbar^2 \varepsilon^2}{m}\right)^{1/3}(n^{2/3} - m^{2/3})$

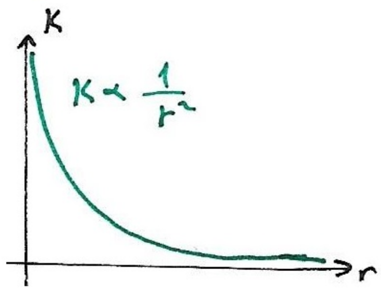
$$\boxed{\omega_{nm} = \frac{3}{2\hbar}\left(\frac{\hbar^2 \varepsilon^2}{m}\right)^{1/3}(n^{2/3} - m^{2/3})}$$

3.2. $Z=1$ ve $n=1$ kabuğu için, yani taban durumunda bulunan bir Hidrojen atomunun boyutu $n=1$ kabuğunun yarıçapı olarak tanımlanır. $\left(a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{me^2}\right)$ Bu temel atomik boyutun doğrudan belirsizlik ilkesi ile elde edilebileceğini gösteriniz.

$$V(r) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Elektron ve çekirdek arasında oluşan potansiyel dağılımı, elektronun çekirdek üzerine çökmesine eğilimli olduğunu gösterir.





Eğer elektron R boyutlu bir bölgede lokalize olmuşsa ; $\Delta x = R$

$$\Delta x \Delta p \geq \hbar \Rightarrow R\Delta p \geq \hbar$$

$$\Delta p = \frac{\hbar}{R}$$

Bu yaklaşım altında elektronun kinetik enerjisi;

$$K = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{2m}\frac{1}{R^2}$$

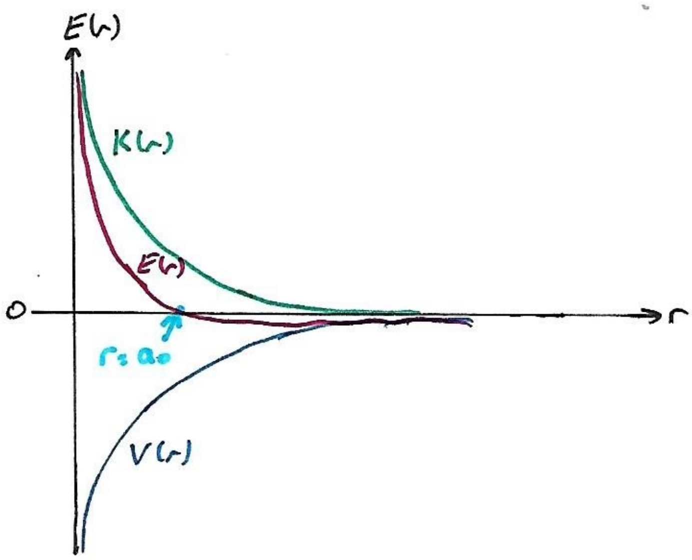
Toplam enerji ifadesi;

$$E = K + V = \frac{\hbar^2}{2m}\frac{1}{R^2} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R}$$

$$\frac{\partial E}{\partial R} = 0 = -\frac{2\hbar^2}{2m}\frac{1}{R^3} + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$$

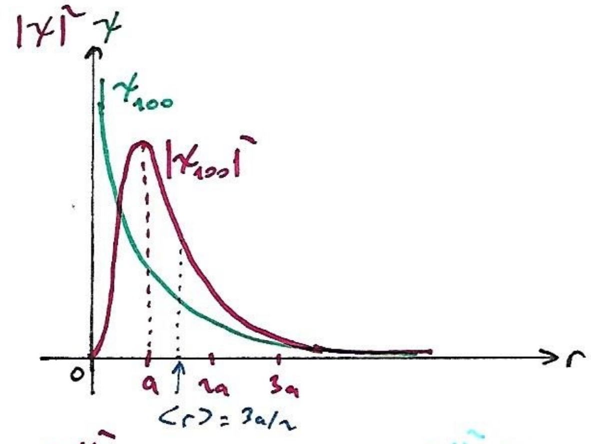
$$0 = -\frac{\hbar^2}{mR} + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{1} \Rightarrow R = \frac{4\pi\varepsilon_0\hbar^2}{me^2} = a_0 = 0{,}529\ \text{Å}$$

Araş . Gör. Çağlar Çetinkaya
caglarcetinkaya@istanbul.edu.tr



Belirsizlik ilkesi atomun minimum momentum ve dolayısıyla minimum kinetik enerji değerini, elektronun çekirdekten olan uzaklığı ile ilgili sıfır nokta enerjisi doğasında verilebileceğini ifade eder.

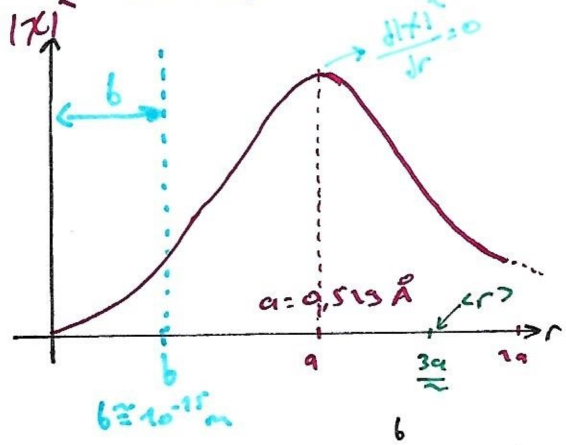
3.3. Hidrojen atomunun taban durumunda bulunan bir elektronun çekirdeğin içinde bulunma olasılığı ne kadardır?



$$\psi_{n\ell m}(r,\theta,\phi) = R_{n\ell}(r)\, Y_\ell^m(\theta,\phi)$$

$$\psi_{100} = R_{10}(r)\, Y_0^0 = \underbrace{\frac{2}{\sqrt{a^3}}\, e^{-r/a}}_{R_{10}}\ \underbrace{\frac{1}{\sqrt{4\pi}}}_{Y_0^0}$$

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}}\, e^{-r/a}$$



Çekirdeğin yarıçapı b ise, elektronun çekirdeğin içinde bulunma olasılığı;

$$P(0<r<b) = \int_0^b \psi_{100}^* \psi_{100}\, dV$$ ile hesaplanır.

$$P(0<r<b) = \frac{1}{\pi a^3} \int_0^b e^{-2r/a}\, r^2 dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$\Rightarrow P(0<r<b) = \frac{4}{a^3} \int_0^b r^2 e^{-2r/a}\, dr \qquad \int u\, dv = uv - \int v\, du$$

$$u = r^2 \longrightarrow du = 2r\, dr$$

$$dv = e^{-2r/a} dr \longrightarrow v = \left(-\frac{2}{a}\right)^{-1} e^{-2r/a}$$

$$P(0<r<b) = \frac{4}{a^3}\left[\underbrace{r^2}_{u} \underbrace{\left(-\frac{2}{a}\right)^{-1} e^{-2r/a}}_{v} \Big|_0^b - \int_0^b \underbrace{\left(-\frac{2}{a}\right)^{-1} e^{-2r/a}}_{v} \underbrace{2r\,dr}_{du} \right]$$

$$P(0<r<b) = \frac{4}{a^3}\left[\left(-b^2 \left(\frac{2}{a}\right)^{-1} e^{-2b/a} - 0\right) + a \int_0^b r\, e^{-2r/a}\, dr \right]$$

$$u = r \rightarrow du = dr \qquad I = \int u\, dv = uv - \int v\, du$$

$$dv = e^{-2r/a}\, dr \rightarrow v = -\frac{a}{2} e^{-2r/a}$$

$$I = r\left(-\frac{a}{2}\right) e^{-2r/a} \Big|_0^b - \int_0^b \left(-\frac{a}{2}\right) e^{-2r/a}\, dr$$

$$I = \left(-\frac{ab}{2} e^{-2b/a} - 0\right) + \frac{a}{2} \int_0^b e^{-2r/a}\, dr$$

$$I = -\frac{ab}{2} e^{-2b/a} - \frac{a^2}{4}\left(e^{-2r/a}\right)\Big|_0^b = -\frac{ab}{2} e^{-2b/a} - \frac{a^2}{4}\left(e^{-2b/a} - 1\right)$$

$$I = +\frac{a^2}{4} - e^{-2b/a}\left(\frac{a^2}{4} + \frac{ab}{2}\right)$$

$$\Rightarrow P(0<r<b) = \frac{4}{a^3}\left[-\frac{ab^2}{2} e^{-2b/a} + a\left(\frac{a^2}{4} - e^{-2b/a}\left(\frac{a^2}{4} + \frac{ab}{2}\right)\right)\right]$$

$$P(0<r<b) = -\frac{2b^2}{a^2} e^{-2b/a} + 1 - e^{-2b/a} - \frac{2b}{a} e^{-2b/a}$$

$$P(0<r<b) = 1 - \left(1 + \frac{2b}{a} + \frac{2b^2}{a^2}\right) e^{-2b/a}$$

$b = 10^{-15}\ \text{m} = \text{fm}$

$a = 0{,}529\ \text{Å} = 0{,}529 \times 10^{-10}\ \text{m}$

$$\Rightarrow P(0<r<10^{-15}\,\text{m}) \cong \frac{4}{3}\left(\frac{10^{-15}}{0{,}529\times 10^{-10}}\right)^3 = 1{,}07\times 10^{-14} \rightarrow P(0<r<10^{-15}) \cong \%\,10^{-12} \;//$$

3.4. Hidrojen atomunun 1. uyarılmış durumunda bulunan bir elektronun,

a) Dalga fonksiyonunu oluşturunuz,

b) Bu durumda elektronun bulunabileceği en olası r değeri nedir?

c) $\langle r^s \rangle$ 'yi hesaplayınız ve $s=1$ için $\langle r \rangle$ 'yi ve $s=2$ için $\langle r^2 \rangle$ 'yi hesaplayınız.

d) Δr değerini hesaplayınız.

$R_{nl}(r)$.

$$R_{10} = 2a^{-3/2}\exp(-r/a)$$

$$R_{20} = \frac{1}{\sqrt{2}}a^{-3/2}\left(1-\frac{1}{2}\frac{r}{a}\right)\exp(-r/2a)$$
$$R_{21} = \frac{1}{\sqrt{24}}a^{-3/2}\frac{r}{a}\exp(-r/2a)$$

$$R_{30} = \frac{2}{\sqrt{27}}a^{-3/2}\left(1-\frac{2}{3}\frac{r}{a}+\frac{2}{27}\left(\frac{r}{a}\right)^2\right)\exp(-r/3a)$$
$$R_{31} = \frac{8}{27\sqrt{6}}a^{-3/2}\left(1-\frac{1}{6}\frac{r}{a}\right)\left(\frac{r}{a}\right)\exp(-r/3a)$$
$$R_{32} = \frac{4}{81\sqrt{30}}a^{-3/2}\left(\frac{r}{a}\right)^2\exp(-r/3a)$$

$$R_{40} = \frac{1}{4}a^{-3/2}\left(1-\frac{3}{4}\frac{r}{a}+\frac{1}{8}\left(\frac{r}{a}\right)^2-\frac{1}{192}\left(\frac{r}{a}\right)^3\right)\exp(-r/4a)$$
$$R_{41} = \frac{\sqrt{5}}{16\sqrt{3}}a^{-3/2}\left(1-\frac{1}{4}\frac{r}{a}+\frac{1}{80}\left(\frac{r}{a}\right)^2\right)\frac{r}{a}\exp(-r/4a)$$
$$R_{42} = \frac{1}{64\sqrt{5}}a^{-3/2}\left(1-\frac{1}{12}\frac{r}{a}\right)\left(\frac{r}{a}\right)^2\exp(-r/4a)$$
$$R_{43} = \frac{1}{768\sqrt{35}}a^{-3/2}\left(\frac{r}{a}\right)^3\exp(-r/4a)$$

$Y_l^m(\theta,\phi)$.

$$Y_0^0 = \left(\frac{1}{4\pi}\right)^{1/2} \qquad Y_2^{\pm 2} = \left(\frac{15}{32\pi}\right)^{1/2}\sin^2\theta e^{\pm 2i\phi}$$
$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2}\cos\theta \qquad Y_3^0 = \left(\frac{7}{16\pi}\right)^{1/2}(5\cos^3\theta - 3\cos\theta)$$
$$Y_1^{\pm 1} = \mp\left(\frac{3}{8\pi}\right)^{1/2}\sin\theta e^{\pm i\phi} \qquad Y_3^{\pm 1} = \mp\left(\frac{21}{64\pi}\right)^{1/2}\sin\theta(5\cos^2\theta - 1)e^{\pm i\phi}$$
$$Y_2^0 = \left(\frac{5}{16\pi}\right)^{1/2}(3\cos^2\theta - 1) \qquad Y_3^{\pm 2} = \left(\frac{105}{32\pi}\right)^{1/2}\sin^2\theta\cos\theta e^{\pm 2i\phi}$$
$$Y_2^{\pm 1} = \mp\left(\frac{15}{8\pi}\right)^{1/2}\sin\theta\cos\theta e^{\pm i\phi} \qquad Y_3^{\pm 3} = \mp\left(\frac{35}{64\pi}\right)^{1/2}\sin^3\theta e^{\pm 3i\phi}$$

$$Y_l^m(\theta,\phi) = \epsilon\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}e^{im\phi}P_l^m(\cos\theta),$$

$$\int_0^{2\pi}\int_0^{\pi}[Y_l^m(\theta,\phi)]^*[Y_{l'}^{m'}(\theta,\phi)]\sin\theta\, d\theta\, d\phi = \delta_{ll'}\delta_{mm'}.$$

$$\int \psi_{nlm}^*\psi_{n'l'm'}r^2\sin\theta\, dr\, d\theta\, d\phi = \delta_{nn'}\delta_{ll'}\delta_{mm'}.$$

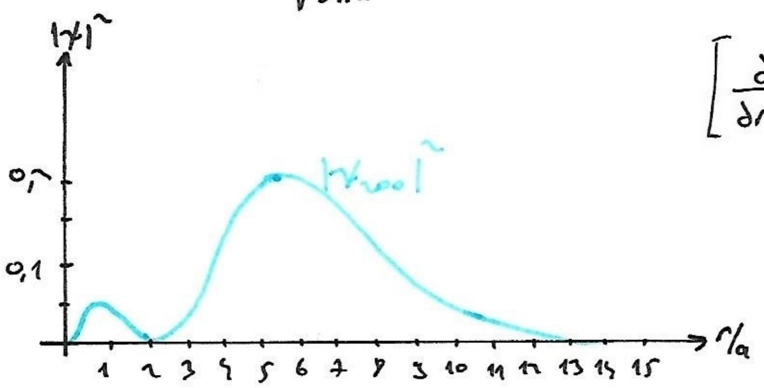
a: $\psi_{nlm} = R_{nl} Y_l^m$

1. uyarılmış durum: $n=2,\ l=0,\ m=0$

$$R_{20} = \frac{1}{\sqrt{2a^3}}\left(1-\frac{r}{2a}\right)e^{-r/2a}$$
$$Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

$$\Rightarrow \psi_{200}(r) = \frac{1}{\sqrt{8\pi a^3}}\left(1-\frac{r}{2a}\right)e^{-r/2a} \Rightarrow |\psi_{200}|^2 = \frac{1}{8\pi a^3}\left(1-\frac{r}{2a}\right)^2 e^{-r/a}\, r^2$$

$$\left[\frac{\partial}{\partial r}|\psi_{200}|^2 r^2\right] = 0 \Rightarrow \frac{\partial P}{\partial r} = 0$$



$$\int_{r=0}^{\infty}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\psi^*\psi\, dV = 1$$

$$1 = \iiint \underbrace{\psi^*\psi\, r^2}_{P}\sin\theta\, dr\, d\theta\, d\phi$$

Araş . Gör. Çağlar Çetinkaya
caglarcetinkaya@istanbul.edu.tr

$$b: \int_{r=0}^{\infty}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \underbrace{\frac{1}{8\pi a^3}\left(1-\frac{r}{2a}\right)^2 e^{-r/a} r^2}_{P} \sin\theta \, dr \, d\theta \, d\phi$$

$$\Rightarrow 0 = \frac{d}{dr}\left[\frac{1}{8\pi a^3}\left(1-\frac{r}{2a}\right)^2 e^{-r/a} r^2\right]$$

$$0 = \frac{d}{dr}\left[\left(1+\frac{r^2}{4a^2}-\frac{r}{a}\right)e^{-r/a} r^2\right] \Rightarrow 0 = \frac{d}{dr}\left[r^2 e^{-r/a} + \frac{r^4}{4a^2}e^{-r/a} - \frac{r^3}{a}e^{-r/a}\right]$$

$$0 = \frac{d}{dr}\left[\left(r^2 + \frac{r^4}{4a^2} - \frac{r^3}{a}\right)e^{-r/a}\right]$$

$$0 = \left[\left(2r + \frac{r^3}{a^2} - \frac{3r^2}{a}\right)\cancel{e^{-r/a}} + \left(r^2 + \frac{r^4}{4a^2} - \frac{r^3}{a}\right)\left(-\frac{1}{a}\right)\cancel{e^{-r/a}}\right]$$

$$0 = 2r + \frac{r^3}{a^2} - \frac{3r^2}{a} - \frac{r^2}{a} - \frac{r^4}{4a^3} + \frac{r^3}{a^2}$$

$$0 = 2r + \frac{2r^3}{a^2} - \frac{4r^2}{a} - \frac{r^4}{4a^3} \Rightarrow 2r\left(1 + \frac{r^2}{a^2} - \frac{4r}{2a} - \frac{r^3}{8a^3}\right) = 0$$

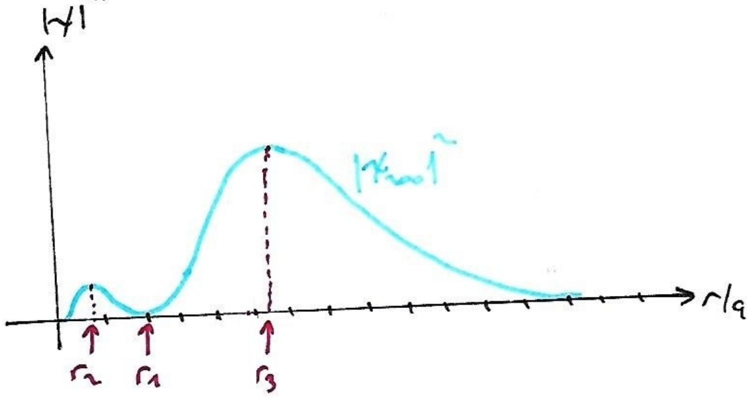
$$0 = 1 + \frac{r^2}{a^2} - \frac{2r}{a} - \frac{r^3}{8a^3}$$

$$\Rightarrow \frac{(2a-r)(4a^2-6ar+r^2)}{8a^3} = 0$$

$$\Rightarrow \quad r_1 = 2a$$

$$r_2 = 3a - \sqrt{5}a = 0{,}76a$$

$$\boxed{r_3 = 3a + \sqrt{5}a = 5{,}24a}$$



Araş . Gör. Çağlar Çetinkaya
caglarcetinkaya@istanbul.edu.tr

c:

$$\langle r^5 \rangle_{200} = \int_0^\infty \psi^*_{200}\, r^5\, \psi_{200}\, r^2 dr \underbrace{\int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi}_{4\pi}$$

$$\langle r^5 \rangle = 4\pi \int_{r=0}^{r=\infty} \frac{1}{8\pi a^3} \left(1 - \frac{r}{2a}\right)^2 e^{-r/a}\, r^5\, r^2 dr = \frac{1}{2a^3} \int_0^\infty r^{5+2} \left(1 - \frac{r}{2a}\right)^2 e^{-r/a} dr$$

$$\langle r^5 \rangle = \frac{1}{2a^3} \int_0^\infty \left[r^{5+2} \left(1 + \frac{r^2}{4a^2} - \frac{r}{a}\right)\right] e^{-r/a} dr$$

$$\langle r^5 \rangle = \frac{1}{2a^3} \int_0^\infty \left[r^{5+2} e^{-r/a} + \frac{r^{5+4}}{4a^2} e^{-r/a} - \frac{r^{5+3}}{a} e^{-r/a} \right] dr$$

$$\langle r^5 \rangle = \frac{1}{2a^3} \left[\underbrace{\int_0^\infty r^{5+2} e^{-r/a} dr}_{I_1} + \frac{1}{4a^2} \underbrace{\int_0^\infty r^{5+4} e^{-r/a} dr}_{I_2} + \left(-\frac{1}{a}\right) \underbrace{\int_0^\infty r^{5+3} e^{-r/a} dr}_{I_3} \right]$$

$$\langle r^5 \rangle = \frac{1}{2a^3} \left[I_1 + \frac{I_2}{4a^2} - \frac{I_3}{a} \right] \qquad I = \int_0^\infty r^n e^{-r/b} = n!\, (b)^{n+1}$$

$$I_2 = \int_0^\infty r^{5+4} e^{-r/a} dr = (5+4)!\, a^{5+5} \qquad I_1 = \int_0^\infty r^{5+2} e^{-r/a} dr = (5+2)!\, a^{5+3}$$

$$I_3 = \int_0^\infty r^{5+3} e^{-r/a} dr = (5+3)!\, a^{5+4}$$

$$\langle r^5 \rangle = \frac{1}{2a^3} \left[(5+2)!\, a^{5+3} + \frac{1}{4a^2} (5+4)!\, a^{5+5} - \frac{1}{a} (5+3)!\, a^{5+4} \right]$$

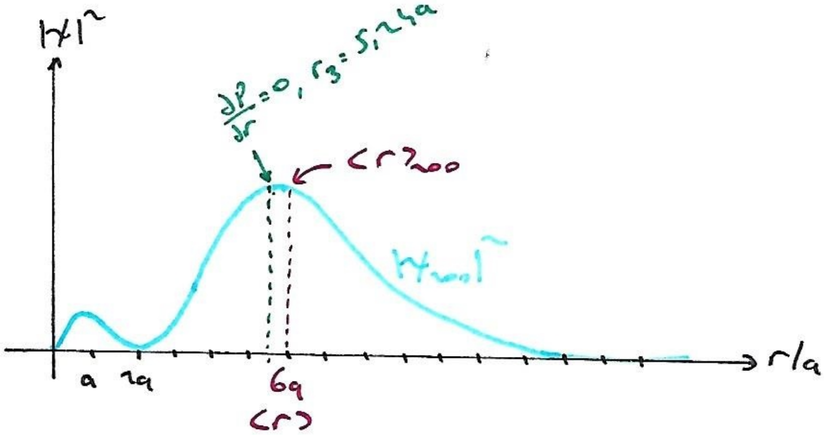
$$\langle r^s \rangle = \frac{1}{2\cancel{a^3}}\left[(s+2)!\, a^s \cancel{a^3} + (s+4)!\, \frac{a^s \cancel{a^3}}{4} - (s+3)!\, a^s \cancel{a^3}\right]$$

$$\langle r^s \rangle = \frac{a^s}{2}\left[(s+2)! - (s+3)! + \frac{(s+4)!}{4}\right]$$

- $s=1 \Rightarrow \langle r \rangle = \frac{a^1}{2}\left[(3!) - (4!) + \frac{5!}{4}\right]$

$$\langle r \rangle = \frac{a}{2}\left[3! - 4.3! + \frac{1}{4}5.4.3!\right] = \frac{a}{2}3!\left(1-4+5\right]$$

$$\langle r \rangle = 3a\left(2\right) \Rightarrow \langle r \rangle = 6a //$$



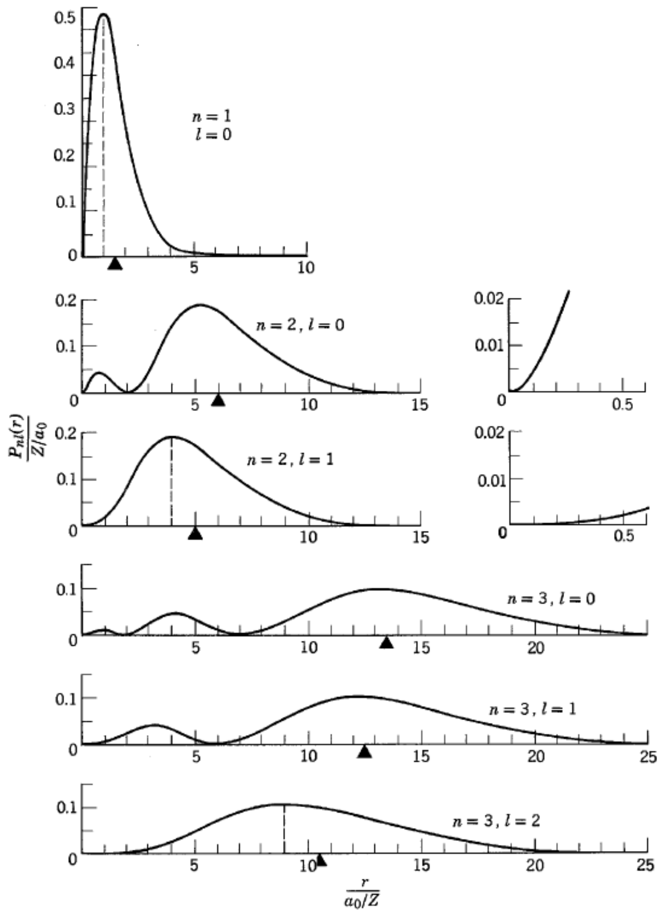
- $s=2 \Rightarrow \langle r^2 \rangle = \left[\frac{a^2}{2}\left(4! - 5! + \frac{6!}{4}\right)\right]$

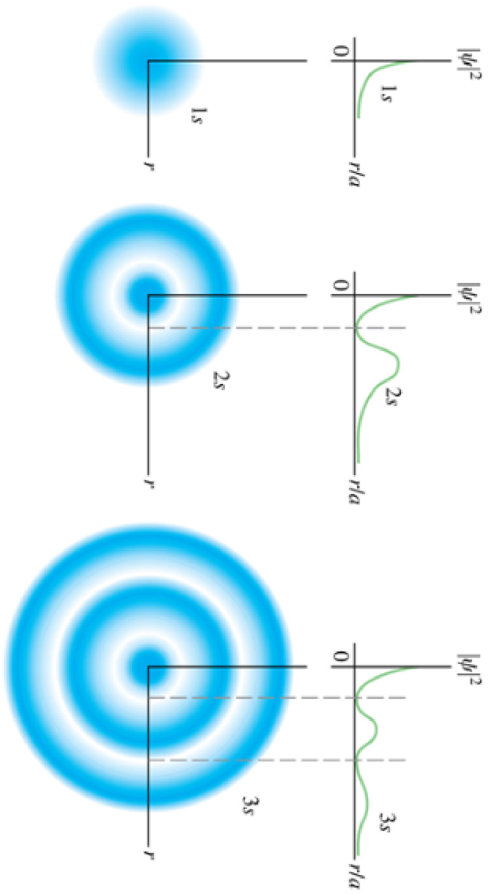
$$\langle r^2 \rangle = \frac{a^2}{2}\left[4! - 5.4! + \frac{1}{4}6.5.4!\right] = \frac{a^2}{2}4!\left(1-5+\frac{15}{2}\right)$$

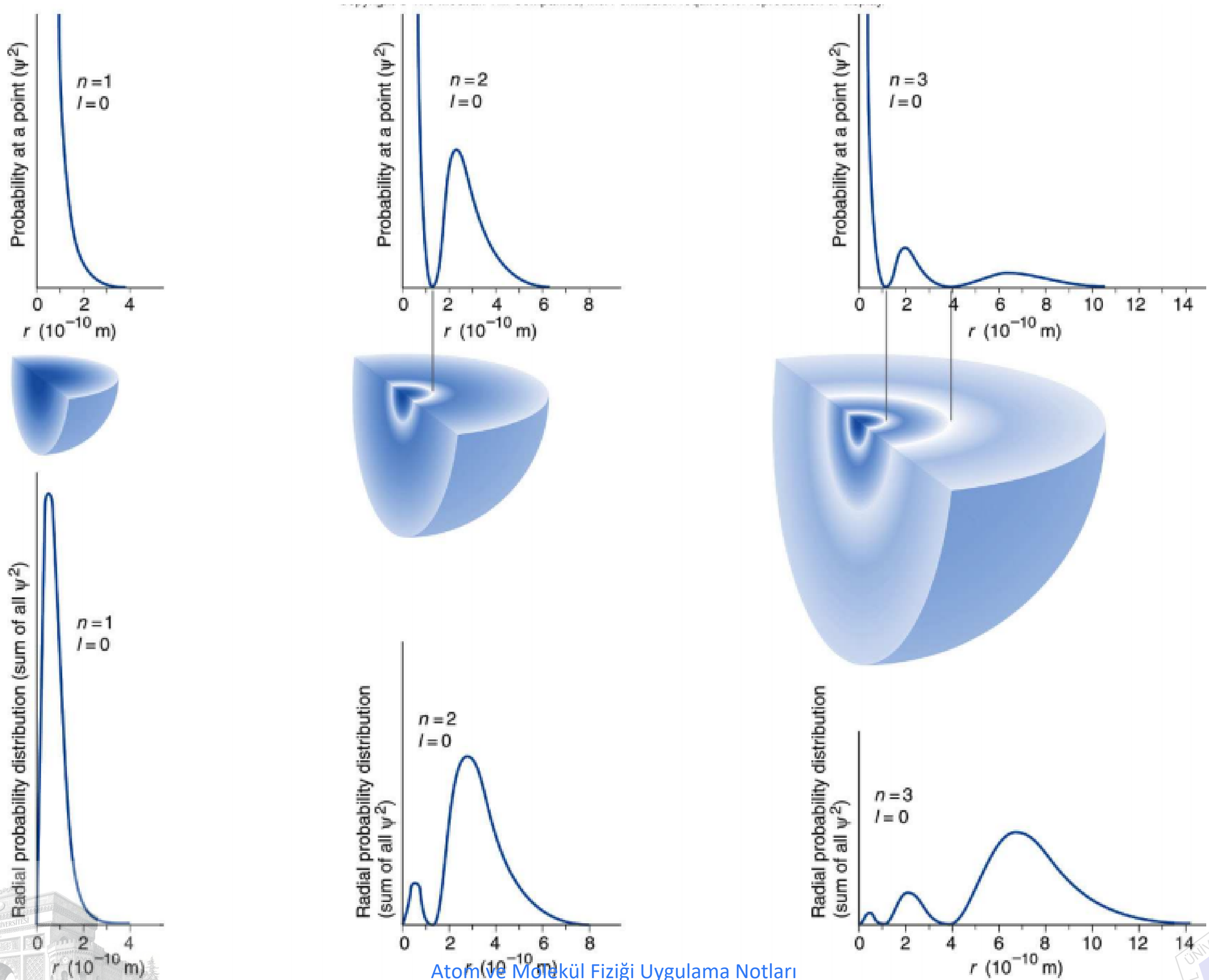
$$\langle r^2 \rangle = 12a^2\left(\frac{7}{2}\right) \Rightarrow \langle r^2 \rangle = 42a^2 //$$

c: $\Delta r = \left[\langle r^2 \rangle - \langle r \rangle^2\right]^{1/2} = \left[42a^2 - 36a^2\right]^{1/2} = \sqrt{6}\,a$

$$\Delta r = a\sqrt{6} //$$


n = 1
l = 0
n = 2, l = 0
n = 2, l = 1
n = 3, l = 0
n = 3, l = 1
n = 3, l = 2
$\frac{P_{nl}(r)}{Z/a_0}$
$\frac{r}{a_0/Z}$


1s
2s
3s
$|\psi|^2$
r
r/a
0

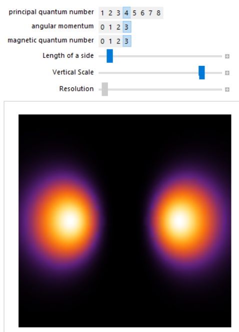

Probability at a point (ψ^2)
n = 1
l = 0
n = 2
l = 0
n = 3
l = 0
r (10^{-10} m)
Radial probability distribution (sum of all ψ^2)

3.5. Hidrojen atomunun $n=4$, $l=3$, $m=3$ durumu için dalga fonksiyonunu kuralım ve $L_x^2+L_y^2$ gözlenebilirini bir şekilde ölçebildiğimizi düşünelim. Hangi değerler hangi olasılıklar ile elde edilir?

$$\psi_{nlm} = R_{nl}(r)\, Y_l^m(\theta,\phi) \qquad R_{43}(r) = \frac{1}{768\sqrt{35}}\frac{1}{\sqrt{a^3}}\left(\frac{r}{a}\right)^3 e^{-r/4a}$$

$$\psi_{433}(r,\theta,\phi) = R_{43}(r)\, Y_3^3(\theta,\phi) \qquad Y_3^3(\theta,\phi) = \left(-\sqrt{\frac{35}{64\pi}}\sin^3\theta\cos\theta\, e^{3i\phi}\right)$$

$$\psi_{433}(r,\theta,\phi) = -\frac{1}{6144\sqrt{\pi}}\frac{1}{a^{9/2}} r^3 e^{-r/4a}\sin^3\theta\, e^{3i\phi}$$



$$L\,\psi_{nlm}(r,\theta,\phi) = \hbar\sqrt{l(l+1)}\,\psi_{nlm}(r,\theta,\phi)$$

$$L_z\,\psi_{nlm}(r,\theta,\phi) = m\hbar\,\psi_{nlm}(r,\theta,\phi)$$

$$|\psi_{nlm}\rangle = |n\rangle|lm\rangle \Rightarrow L|n\rangle|lm\rangle = \hbar\sqrt{l(l+1)}\,|n\rangle|lm\rangle$$

$$L_z|n\rangle|lm\rangle = m\hbar\,|n\rangle|lm\rangle$$

$|l,m\rangle$ özdurumları, dolayısıyla $|n\rangle|l,m\rangle$ yani ψ_{nlm} $\hat{L}_z$ ve $\hat{L}$ operatörlerinin özfonksiyonlarıdır. Bu yüzden L_x ve L_y doğrudan ψ_{nlm} üzerine uygulanamaz. Ancak $L_x^2+L_y^2$ operatörü L_z ve L cinsinden yazılabilir:

$$L^2 = L_x^2 + L_y^2 + L_z^2 \Rightarrow L_x^2 + L_y^2 = L^2 - L_y^2 \;//$$

- $$(L_x^2 + L_y^2)\,\psi_{nlm} = (L^2 - L_y^2)\,\psi_{nlm}$$

$$= L^2\psi_{nlm} - L_z^2\psi_{nlm} \qquad L_z^2\psi_{nlm} = L_z(L_z\psi_{nlm})$$

$$= \left[\hbar^2 l(l+1) - (m\hbar)^2\right]\psi_{nlm} \qquad = L_z(m\hbar\,\psi_{nlm})$$

$$= m\hbar\,(L_z\psi_{nlm})$$

$$(L_x^2 + L_y^2)\,\psi_{433} = (L^2 - L_z^2)\,\psi_{433} \qquad = (m\hbar)^2\psi_{nlm}$$

Araş . Gör. Çağlar Çetinkaya
caglarcetinkaya@istanbul.edu.tr

$$(L_x^2+L_y^2)\,\psi_{433} = L^2\psi_{433} - L_z^2\,\psi_{433}$$

$$= \left[\hbar^2\, 3(3+1) - (3\hbar)^2\right]\psi_{433}$$

$$= 12\hbar^2 - 9\hbar^2$$

$= 3\hbar^2$ %100 olasılıkla. Çünkü etkilenen ψ_{433} durağan durumundadır. Yani L ve L_z'nin özfonksiyonlarında birliktedir.

2. yol; $L_x = \dfrac{L_+ + L_-}{2}$, $L_y = \dfrac{L_+ - L_-}{2i}$ $\qquad [L_+, L_-] \neq 0$

$$L_\pm Y_l^m = \hbar\sqrt{l(l+1) - m(m\pm1)}\; Y_l^{m\pm1}$$

$$\cdot\,(L_x^2+L_y^2)\,\psi_{nlm} = \left[\left(\frac{L_+^2 + L_+L_- + L_-L_+ + L_-^2}{4}\right) + \left(\frac{L_+^2 - L_+L_- - L_-L_+ + L_-^2}{-4}\right)\right]\psi_{nlm}$$

$$(L_x^2+L_y^2)\,\psi_{433} = \frac{1}{4}\left[\cancel{L_+^2} + L_+L_- + L_-L_+ + \cancel{L_-^2} - \cancel{L_+^2} + L_+L_- + L_-L_+ - \cancel{L_-^2}\right]\psi_{433}$$

$$= \frac{1}{2}\left[L_+L_- + L_-L_+\right]\psi_{433}$$

$$\hbar\sqrt{3.4 - 3(3+1)} = 0$$

$$= \frac{1}{2}\left[L_+L_-\,\psi_{433} + \underbrace{L_-L_+\,\psi_{433}}_{0}\right]$$

$$L_+\psi_{433} = R_{43}\,L_+ Y_3^3 = R_{43}\,0 = 0$$

$$= \frac{1}{2}\left[L_+L_-\,\psi_{433}\right]$$

$$= \frac{1}{2}L_+\left(R_{43}\,L_- Y_3^3\right) = \frac{1}{2}L_+\left(R_{43}\,\hbar\sqrt{3.4 - 3.2}\;Y_3^2\right)$$

$$= \frac{L_+}{2}R_{43}\sqrt{6}\,\hbar\, Y_3^2 = \frac{\sqrt{6}\,\hbar}{2}R_{43}\,L_+ Y_3^2$$

$$= \frac{\sqrt{6}\,\hbar}{2}R_{43}\,\hbar\sqrt{3.4 - 2.(2+1)}\;Y_3^3 = \frac{6\hbar^2}{2}R_{43}\,Y_3^3$$

$$= 3\hbar^2\,\psi_{433} \implies (L_x^2+L_y^2)\,\psi_{433} = 3\hbar^2\,\psi_{433}$$