Ad ve Soyad :
Numara :
Bölüm :

Soru:	1	2	3	4	5	Toplam
Türü	Z	Z	Z	S	S	
Puan:						

Atom ve Molekül Fiziği

2018-2019 Bahar Dönemi Örgün Öğretim Ara Sınavı

1. Hartree Teorisi'ni ele alarak ayrıntılı olarak açıklayınız.

2. Bohr Atom Modeli'ni ele alarak,

a. Bohr postülatlarını yazınız ve Bohr modeline göre bir atomun toplam enerjisinin

$$E_n = -\frac{m_e}{2\hbar^2}\left(\frac{ze^2}{4\pi\varepsilon_0}\right)^2\frac{1}{n^2}, \quad n = 1,2,3,..$$

olduğunu gösteriniz.

b. Bohr'un *Karşılık Gelme* ilkesini, Hidrojen atomunun klasik limitteki ışımasına uygulayınız. Yani $n \to \infty$ için elektronun dolanma frekansının, $n_i - n_s = 1$ olmak üzere, n_i yörüngesinden n_s yörüngesine geçiş frekansına yaklaştığını gösteriniz.

3. Atom numarası $Z = 30$ olan Çinko elementinin (${}_{30}Zn$) taban durumda ve birinci uyarılmış durumda,

a. Elektronik konfigürasyonunu belirleyiniz ve mümkün spektral terimlerini bulunuz. Her iki durum için minimum enerjiye sahip spektral terimi belirleyiniz.

b. Enerji diyagramını çiziniz ve spin-yörünge ($\vec{S}\,\vec{L}$ coupling) etkileşimi altında enerjide meydana gelen ΔE_{SL} yarılmalarını gösteriniz.

c. Uyarılmış durumda meydana gelen spin-yörünge etkileşiminden kaynaklı yarılmaların $\Delta E_{SL} = -A\,\vec{S}.\vec{L}$ enerjisinde olduğunu varsayarak (A:sabit), herbir enerjideki yarılmayı A cinsinden hesaplayınız.

4. Hidrojen atomu için spin-yörünge etkileşiminden kaynaklı enerji özdeğerlerine gelen 1. dereceden düzeltme terimi aşağıdaki gibidir:

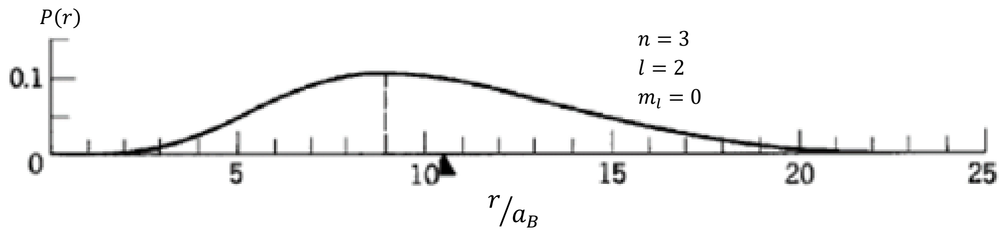
$$\Delta E_{SL} =< H'_{SL} >= \frac{1}{2(m_e c)^2} < \frac{1}{r}\frac{\partial U(r)}{\partial r}\,\vec{S}.\vec{L} >$$

$\psi_{2,1,1}$ durumuna uyarılmış bir elektron için ΔE_{LS} değerini hesaplayınız.

$$R_{2,1}(r) = \frac{1}{\sqrt{24}}a_B^{-3/2}\frac{r}{a_B}e^{-r/2a_B} \qquad Y_1^1(\theta,\varphi) = -\left(\frac{3}{8\pi}\right)^{1/2}\sin\theta\, e^{+i\varphi} \qquad U(r) = \frac{-Ze^2}{4\pi\varepsilon_0}\frac{1}{r}$$

$$\int_0^\infty r^n e^{-r/b}dr = n!\ (b)^{n+1} \qquad \int \sin\theta\, cos^{(n+1)}\theta\, d\theta = \frac{-cos^{(n+1)}\theta}{(n+1)} \qquad \int_0^\pi sin^3\theta\, d\theta = \frac{4}{3}$$

5. Hidrojen atomunun $\psi_{3,2,0}$ durumunda bulunan bir elektronun olasılık yoğunluğunun radyal değişimi aşağıda verilmektedir.



$\psi_{3,2,0}$ durumunda bulunan elektronun,

a. Bulunabileceği <u>en olası</u> r değerini Bohr yarıçapı (a_B) cinsinden hesaplayınız.

b. Açısal momentumunun z yönündeki bileşeninin beklenen değeri, $< L_z >$'yi hesaplayınız.

c. Açısal momentumunun karesinin beklenen değeri, $< L^2 >$'yi hesaplayınız.

Not: Beklenen değerleri L_z ve L^2 nin özdeğerlerini kullanarak veya L_z ve L^2 operatörlerinin küresel koordinatlarda ifade edilmiş hallerini kullanarak hesaplayabilirsiniz.

$$R_{3,2}(r) = \frac{4}{81\sqrt{30}} a_B^{-\frac{3}{2}} \left(\frac{r}{a_B}\right)^2 e^{-\frac{r}{3a_B}}, \qquad Y_2^0(\theta,\varphi) = \left(\frac{5}{16\pi}\right)^{\frac{1}{2}} (3\cos^2\theta - 1)$$

$$\int_0^\infty r^n e^{-r/b} dr = n!\ (b)^{n+1}, \qquad \int \sin\theta\, cos^n\theta\, d\theta = \frac{-cos^{(n+1)}\theta}{(n+1)}$$

$$L_z = -i\hbar\frac{\partial}{\partial\varphi}, \qquad L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right],$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

Not: 1.2. ve 3. sorular zorunludur. 4. ve 5. sorulardan yalnızca bir tanesi seçilmelidir.

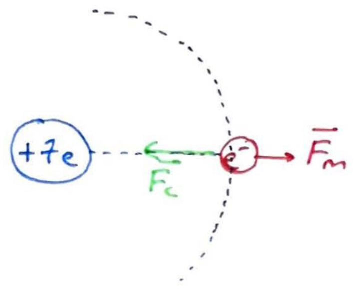
Başarılar 😊

Prof. Dr. Yasemin Akkaya

2)

a) Bohr postülaları;

• Elektron atom çekirdeği etrafında dairesel bir yörüngede dolaşır. Dairesel hareket için gerekli olan merkezcil kuvveti elektron ile çekirdek arasındaki Coulomb kuvveti sağlar.



Coulomb potansiyeli;

$$U(r) = \frac{Ze(-e)}{4\pi\varepsilon_0}\frac{1}{r} = \frac{-Ze^2}{4\pi\varepsilon_0}\frac{1}{r}$$

Coulomb kuvveti;

$$\vec{F}_c = -\vec{\nabla} U(r) = -\frac{d}{dr}\left(\frac{-Ze^2}{4\pi\varepsilon_0}\frac{1}{r}\right)\hat{e}_\rho$$

$$\vec{F}_c = \frac{-Ze^2}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{e}_\rho$$

Merkezcil kuvvet;

$$\vec{F}_m = m_e\frac{v^2}{r}\hat{e}_\rho$$

Net kuvvet; $\vec{F}_{net} = 0 \Rightarrow \vec{F}_c + \vec{F}_m = \vec{0}$

$$-\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{e}_\rho + \frac{m_e v^2}{r}\hat{e}_\rho = \vec{0}$$

$$\boxed{\frac{m_e v^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r^2}} \;\star$$

• Elektronun açısal momentumu ancak $\hbar$ ve $\hbar$'ın tam katları olan dairesel yörüngeler üzerinden tanımlanır. Bu yörüngelere izinli yörüngeler denir:

$$\star\star \;\boxed{m_e v r = n\hbar} \qquad n = 1, 2, 3, \ldots, \infty$$

• Elektron izinli yörüngelerde dolaşırken ivmeli hareket yapmasına rağmen ışıma yapmaz. Sadece belirli bir n_i yörüngesinden n_s yörüngesine geçerken $E_s - E_i = \Delta E$ enerjisine sahip foton yayınlar. (veya soğurur)

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$\star$: $\dfrac{mv^2}{r} = \dfrac{Ze^2}{4\pi\varepsilon}\dfrac{1}{r^2} \Rightarrow \boxed{r = \dfrac{Ze^2}{4\pi\varepsilon_0}\dfrac{1}{mv^2}}$ $\star'$

$\star\star$: $mvr = n\hbar \Rightarrow v = \dfrac{n\hbar}{mr}$

$$v = \frac{n\hbar}{m}r^{-1} = \frac{n\hbar}{m}\left(\frac{4\pi\varepsilon_0}{Ze^2}mv^2\right)$$

$$\boxed{v = \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{n\hbar}} \quad \star''$$

$\star'$: $r = \dfrac{Ze^2}{4\pi\varepsilon_0}\dfrac{1}{m}v^{-2} = \dfrac{Ze^2}{4\pi\varepsilon_0}\dfrac{1}{m}\left[\dfrac{4\pi\varepsilon_0}{Ze^2}n\hbar\right]^2$

$$\boxed{r_n = \frac{4\pi\varepsilon_0}{mZe^2}\hbar^2 n^2} \quad \star'''$$

- $E = T + U = \dfrac{1}{2}mv^2 + \left(\dfrac{-Ze^2}{4\pi\varepsilon}\dfrac{1}{r}\right)$

$v_n = \dfrac{Ze^2}{4\pi\varepsilon}\dfrac{1}{n\hbar}$ $\qquad$ $r_n = \dfrac{4\pi\varepsilon}{mZe^2}\hbar^2 n^2$

$$\Rightarrow E_n = \frac{1}{2}m\left(\frac{Ze^2}{4\pi\varepsilon}\frac{1}{n\hbar}\right)^2 + \left(\frac{Ze^2}{4\pi\varepsilon}\frac{mZe^2}{4\pi\varepsilon}\frac{1}{n^2\hbar^2}\right)$$

$$E_n = \frac{-mZ^2e^4}{(4\pi\varepsilon_0)^2 2\hbar^2}\frac{1}{n^2} \Rightarrow \boxed{E_n = -\frac{m}{2\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon}\right)^2\frac{1}{n^2}}, \quad n = 1, 2, 3, \ldots$$

b) Klasik fizikte elektron dolanma frekansı; ν_k

$$\nu_k = \frac{v_n}{2\pi r_n} = \frac{1}{2\pi r_n}\underbrace{\left(\frac{n\hbar}{mr}\right)}_{v_n} = \frac{n\hbar}{2\pi m}\frac{1}{r_n^2} = \frac{n\hbar}{2\pi m}\underbrace{\left(\frac{mZe^2}{4\pi\varepsilon}\frac{1}{n^2\hbar^2}\right)^2}_{1/r^2}$$

$$\nu_k = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{me^4}{4\pi\hbar^3}\frac{2}{n^3}\right) \Rightarrow \boxed{\nu_k = cR\frac{2}{n^3}}$$

Kuantum fiziğinde elektron $n_i \to n_s$ geçişinde yayınladığı frekans;

$$\nu_{is} = \frac{E_{is}}{h} = \frac{\cancel{h}c}{\cancel{h}\lambda}\left(\frac{1}{n_s^2}-\frac{1}{n_i^2}\right) = cR\left(\frac{1}{n_s^2}-\frac{1}{n_i^2}\right)$$

$$\nu_{is} = cR\left[\frac{n_i^2 - n_s^2}{n_s^2 n_i^2}\right] = cR\left[\frac{\overbrace{(n_i-n_s)}^{1}(n_i+n_s)}{n_s^2 n_i^2}\right]$$

$n_i - n_s = 1$

$n \to \infty \Rightarrow n_i \approx n_s$

$$\nu_{is} = cR\left(\frac{2n}{n^4}\right) \Rightarrow \boxed{\nu_{is} = cR\frac{2}{n^3}}$$

$\underline{\nu_{is} = \nu_k}$

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3) a:

Taban Durum;

$_{30}Zn : 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}$

1s: ↑↓ 2s: ↑↓ 2p: ↑↓ ↑↓ ↑↓ 3s: ↑↓ 3p: ↑↓ ↑↓ ↑↓ 4s: ↑↓ 3d: ↑↓ ↑↓ ↑↓ ↑↓ ↑↓

$L = \sum_{i=1}^{Z=30} (m_\ell)_i = 0 \longrightarrow S$

$S = \sum_{i=1}^{30} s_i = 0 \longrightarrow$ Ç.T. $= 2S+1$, Ç.T. $= 1$

$j : |L-S|, \ldots, (L+S)$ $\qquad {}^{2S+1}L_j$

$j = 0, 0, 0 \longrightarrow j = 0$

${}^1S_0$ minimum enerjiye sahip spektral terim (kapalı kabuk)

1. uyarılmış durum

$_{30}Zn^* : 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^9\ 4p^1$ (e^- 3d → 4p)

1s: ↑↓ 2s: ↑↓ 2p: ↑↓ ↑↓ ↑↓ 3s: ↑↓ 3p: ↑↓ ↑↓ ↑↓ 4s: ↑↓

3d⁹: ↑ $m_\ell=-2$; ↑↓ $m_\ell=-1$; ↑↓ $m_\ell=0$; ↑↓ $m_\ell=1$; ↑↓ $m_\ell=2$

4p¹: — $m_\ell=-1$; — $m_\ell=0$; ↑ $m_\ell=+1$

Dolu orbitaller: $\sum_{1}^{20} (m_\ell)_i = 0$, $\sum_{i=1}^{20} s_i = 0$

$\bullet\ L = \sum_{i=20}^{30} (m_\ell)_i = 2+1+0-1-2+2+1+0-1+1$

$L = 3 \longrightarrow F$

$\bullet\ S = \sum_{i=20}^{30} s_i = 5\left(\frac{1}{2}\right) + 4\left(-\frac{1}{2}\right) + 1\left(\frac{1}{2}\right)$

$S = 1 \longrightarrow$ Ç.T. $= 2S+1 = 3$

$\bullet\ j : |L-S|, \ldots, (L+S) : |3-1|, \ldots (3+1)$

$j : 2, 3, 4$

Spektral terimler;

${}^3F_2,\ {}^3F_3,\ {}^3F_4$

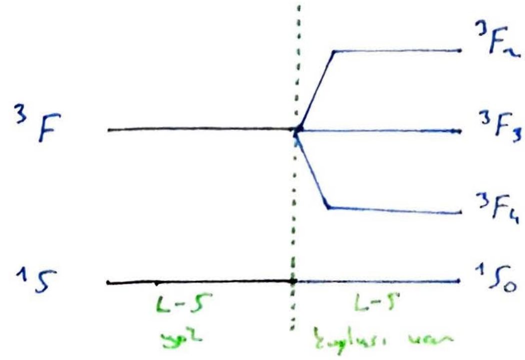
${}^3F_4$: minimum enerjili terim

$3d^9\ 4p^1$ orbitallerin alabileceği elektron sayısı $= 10+6 = 16$

elektron sayısı $= 9+1 = 10$

$16 > 10 \rightarrow \frac{16}{2} = 8 < 10$ orbitallerin yarıdan fazlası dolu, $j=4$ terimi minimum enerjiye sahiptir.

b:



c: $\Delta E_{SL} = -A\ \bar{L}.\bar{S}$

$\bar{J} = \bar{L} + \bar{S} \Rightarrow J^2 = (\bar{L}+\bar{S}).(\bar{L}+\bar{S})$

$J^2 = L^2 + S^2 + 2\bar{L}.\bar{S}$

$\bar{L}.\bar{S} = \frac{1}{2}(J^2 - L^2 - S^2)$

$\Delta E_{SL} = \frac{-A}{2}(J^2 - L^2 - S^2)$

Uygulanmış durum; ^3F $\quad l=3$, $s=1$

$$\Delta E_{sL} = \frac{-A}{2}\left(j^2 - L^2 - S^2\right)$$

$$\Delta E_{sL} = \frac{-A\hbar^2}{2}\left(j(j+1) - L(L+1) - s(s+1)\right)$$

$$\Delta E_{sL} = \frac{-A\hbar^2}{2}\left[j(j+1) - 3(3+1) - 1(1+1)\right]$$

$$\Delta E_{sL}(j) = \frac{-A\hbar^2}{2}\left[j(j+1) - 14\right]$$

- $^3F_2 \rightarrow j=2$

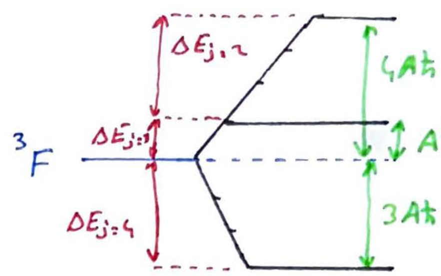
$$\Delta E_{sL}(j=2) = \frac{-A\hbar^2}{2}\left[2(2+1)-14\right] = +4A\hbar^2$$

- $^3F_3 \rightarrow j=3$

$$\Delta E_{sL}(j=3) = \frac{-A\hbar^2}{2}\left[3(3+1)-14\right] = +A\hbar$$

- $^3F_4 \rightarrow j=4$

$$\Delta E_{sL}(j=4) = \frac{-A\hbar^2}{2}\left[4(4+1)-14\right] = -3A\hbar^2$$



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4) $\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)\, Y_l^{m_l}(\theta,\phi)$

$$\psi_{211}(r,\theta,\phi) = R_{21}(r)\, Y_1^1(\theta,\phi) = \underbrace{\frac{1}{\sqrt{24}}\, a^{-3/2}\, \frac{r}{a}\, e^{-r/2a}}_{R_{21}} \underbrace{\left[-\left(\frac{3}{8\pi}\right)^{1/2} \sin\theta\, e^{+i\phi}\right]}_{Y_1^1}$$

$$\psi_{211}(r,\theta,\phi) = -\left(\frac{3}{192\pi}\right)^{1/2} a^{-3/2}\, \frac{r}{a}\, \sin\theta\, e^{i\phi}\, e^{-r/2a}$$

$$\Delta E_{SL} = \langle H'_{SL} \rangle = \frac{1}{2(mc)^2} \left\langle \frac{1}{r}\frac{dU(r)}{dr}\, \bar{S}\cdot\bar{L} \right\rangle$$

• $$\frac{dU(r)}{dr} = \frac{d}{dr}\left[\frac{-e^2}{4\pi\varepsilon_0}\frac{1}{r}\right] = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^2}$$

• $$\bar{S}\cdot\bar{L} \Rightarrow \bar{J} = \bar{L}+\bar{S} \Rightarrow J^2 = (\bar{L}+\bar{S})\cdot(\bar{L}+\bar{S}) = L^2 + S^2 + 2\bar{L}\cdot\bar{S}$$

$$\bar{S}\cdot\bar{L} = \frac{1}{2}\left[J^2 - L^2 - S^2\right]$$

$$\bar{S}\cdot\bar{L} = \frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - s(s+1)\right]$$

$$\Rightarrow \Delta E_{SL} = \frac{1}{2(mc)^2}\left\langle \frac{1}{r} \underbrace{\left(\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^2}\right)}_{\frac{du}{dr}} \underbrace{\frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - s(s+1)\right]}_{\bar{S}\cdot\bar{L}} \right\rangle$$

$$\Delta E_{SL} = \frac{\hbar^2}{4(mc)^2}\frac{e^2}{4\pi\varepsilon_0}\left\langle\frac{1}{r^3}\right\rangle \left\langle\left[j(j+1) - l(l+1) - s(s+1)\right]\right\rangle$$

• $\langle \psi_{211} | \left[j(j+1) - l(l+1) - s(s+1)\right] \psi_{211} \rangle$ $\psi_{211} \Rightarrow n=2,\ l=1,\ m_l=1$

p

— $m_l=-1$

— $m_l=0$

$\uparrow$ $m_l=1$

$S=1/2$

$j = |l-s|, \ldots, (l+s) = |1-1/2|, \ldots, (1+1/2)$

$j:\ 1/2,\ 3/2$

$= \langle \psi_{211} | \left[j(j+1) - 1(1+1) - \frac{1}{2}(1/2+1)\right] \psi_{211} \rangle$

$= \langle \psi_{211} | \left[j(j+1) - \frac{11}{4}\right] | \psi_{211} \rangle$

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$j = 1/2 \Rightarrow$

$$\langle \psi_{211} | \left[j(j+1) - \frac{11}{4} \right] | \psi_{211} \rangle = \langle \psi_{211} | \left[\frac{1}{2}(1/2+1) - \frac{11}{4} \right] \psi_{211} \rangle$$

$$= -\frac{8}{4} \underbrace{\langle \psi_{211} | \psi_{211} \rangle}_{1} = -\frac{8}{4} = -2$$

$j = 3/2 \Rightarrow$

$$\langle \psi_{211} | \left[j(j+1) - \frac{11}{4} \right] \psi_{211} \rangle = \langle \psi_{211} | \left[\frac{3}{2}\left(\frac{3}{2}+1\right) - \frac{11}{4} \right] \psi_{211} \rangle$$

$$= \frac{4}{4} \underbrace{\langle \psi_{211} | \psi_{211} \rangle}_{1} = 1$$

- $\langle \frac{1}{r^3} \rangle = \langle \psi_{211} | \frac{1}{r^3} | \psi_{211} \rangle$

$$\langle \frac{1}{r^3} \rangle = + \frac{3}{192\pi} a^{-3} \frac{1}{a^2} \int_V \frac{r^2 \sin^2\theta \overbrace{\left(e^{i\phi} e^{-i\phi} \right)}^{e^0 = 1}}{r^3} e^{-r/a} \underbrace{r^2 \sin\theta \, dr \, d\theta \, d\phi}_{dV}$$

$$\langle \frac{1}{r^3} \rangle = \frac{3}{192\pi} \frac{1}{a^5} \underbrace{\int_0^\infty r e^{-r/a} dr}_{1! (a)^{1+1}} \underbrace{\int_0^\pi \sin^3\theta \, d\theta}_{4/3} \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$$\langle \frac{1}{r^3} \rangle = \frac{3}{192\pi} \frac{1}{a^5} a^2 \frac{4}{3} 2\pi = \frac{8}{192} \frac{1}{a^3}$$

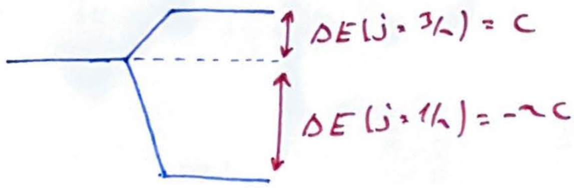
$$\Rightarrow \Delta E_{SL} = \frac{\hbar^2}{4(mc)^2} \frac{e^2}{4\pi\varepsilon_0} \underbrace{\left(\frac{8}{192} \frac{1}{a^3} \right)}_{\langle \frac{1}{r^3} \rangle_{211}} \left(\langle \psi_{211} | \left[j(j+1) - \frac{11}{4} \right] | \psi_{211} \rangle \right)$$

$$\Delta E_{SL} = \underbrace{\frac{1}{384} \left(\frac{\hbar e}{mca} \right)^2 \frac{1}{\pi a \varepsilon_0}}_{C} \langle \psi_{211} | \left[j(j+1) - \frac{11}{4} \right] \psi_{211} \rangle$$

$$\Delta E_{SL} = C \langle \psi_{211} | \left[j(j+1) - \frac{11}{4} \right] \psi_{211} \rangle$$

$$\Delta E_{SL}(j=1/2) = -\frac{8C}{4} = -2C$$

$$\Delta E_{SL}(j=3/2) = +C$$



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5) a)

$$\int |\psi_{320}|^2 dV = \int |R_{32}(r)\, Y_2^0(\theta,\phi)|^2 r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \int |R_{32}|^2 r^2 dr \underbrace{\int |Y_2^0|^2 \sin\theta\, d\theta\, d\phi}_{1} \qquad \begin{array}{l} 0<\theta<\pi \\ 0<\phi<2\pi \end{array}$$

$$\int Y_{l'}^{m'*} Y_l^m \, d\Omega = \delta_{ll'}\delta_{mm'}, \qquad d\Omega \rightarrow \sin\theta\, d\theta\, d\phi$$

$$\int |\psi_{320}|^2 dV = \int \underbrace{|R_{32}|^2 r^2}_{\rho(r)} dr$$

$\rho(r)$: r'ye bağlı olasılık yoğunluğu

$$\Rightarrow \rho(r) = \underbrace{\left[\frac{4}{81\sqrt{30}} a^{-3/2} \left(\frac{r}{a}\right)^2 e^{-r/3a}\right]^2}_{|R_{32}|^2} r^2$$

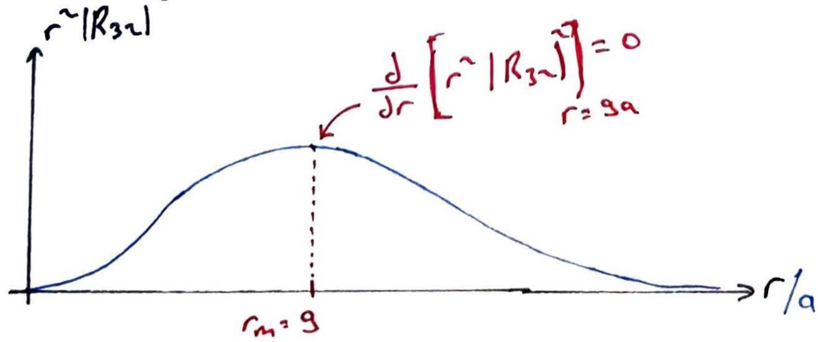
$$= \left(\frac{4}{81\sqrt{30}} a^{-3/2} \frac{1}{a^2}\right)^2 r^4 e^{-2r/3a} r^2$$

$$\rho(r) = \left(\frac{4}{81\sqrt{30}} a^{-3/2} \frac{1}{a^2}\right)^2 r^6 e^{-2r/3a}$$

$$\Rightarrow \left[\frac{d}{dr}\rho(r)\right]_{r=r_{max}} = 0 \Rightarrow \frac{d}{dr}\left[\left(\frac{4}{81\sqrt{30}} a^{-3/2} \frac{1}{a^2}\right)^2 r^6 e^{-2r/3a}\right]_{r=r_{max}=r_m} = 0$$

$$\Rightarrow \left(\frac{4}{81\sqrt{30}} a^{-3/2} \frac{1}{a^2}\right)\left[6 r_m^5 e^{-2r_m/3a} + r_m^6\left(-\frac{2}{3a}\right) e^{-2r_m/3a}\right] = 0$$

$$\Rightarrow 6 r_m^5 + r_m^6\left(\frac{-2}{3a}\right) = 0 \Rightarrow 6r^5 = \frac{2r^6}{3a} \Rightarrow r_m = 9a$$



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b)

1. yol : $L_z \psi_{nlm_l} = \underbrace{m_l \hbar}_{} \psi_{nlm_l}$

$L_z = -i\hbar \frac{\partial}{\partial\phi}$ operatörünün özdeğerleri

$$\psi_{320} \Rightarrow \left.\begin{matrix} n=3 \\ l=2 \\ m_l=0 \end{matrix}\right\} \quad L_z \psi_{320} = \underbrace{0\hbar}_{} \psi_{320}$$

$$\bullet \; \langle L_z \rangle = \langle \psi_{320} | L_z | \psi_{320} \rangle = \langle \psi_{320} | 0\hbar | \psi_{320} \rangle = 0\hbar \langle \psi_{320} | \psi_{320} \rangle$$

$$\langle L_z \rangle = 0\hbar \,//$$

2. yol :

$$\psi_{320}(r,\theta,\phi) = R_{32}(r)\, Y_2^0(\theta)$$

$$\psi_{320} = \underbrace{\frac{4}{81\sqrt{30}} a^{-3/2} \left(\frac{r}{a}\right)^2 e^{-r/3a}}_{R_{32}(r)} \underbrace{\sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1)}_{Y_2^0(\theta)}$$

$$\bullet \; \langle L_z \rangle = \langle \psi_{320} | \underbrace{\left(-i\hbar \frac{\partial}{\partial\phi}\right)}_{L_z} | \psi_{320} \rangle = 0 \qquad \psi_{320} \;\; \phi\text{'ye bağlı değil}$$

$$\langle L_z \rangle = 0 \,//$$

c)

1. yol : $L^2 |\psi_{nlm_l}\rangle = \underbrace{\hbar^2 l(l+1)}_{} |\psi_{nlm_l}\rangle$

L^2 operatörünün özdeğerleri

$$\bullet \; \langle L^2 \rangle = \langle \psi_{320} | L^2 | \psi_{320} \rangle = \langle \psi_{320} | \hbar^2 2(2+1) | \psi_{320} \rangle = 6\hbar^2 \underbrace{\langle \psi_{320} | \psi_{320} \rangle}_{1}$$

$$\langle L^2 \rangle = 6\hbar^2 \,//$$

2. yol :

$$L^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right]$$

$$\bullet \; \langle L^2 \rangle = \langle \psi_{320} | \left[-\hbar^2 \left(\frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right) \right] | \psi_{320} \rangle$$

$$\langle L^2\rangle = -\hbar^2\left[\langle \psi_{320}|\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)|\psi_{320}\rangle + \langle\psi_{320}|\underbrace{\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}|\psi_{320}\rangle}_{\frac{1}{\sin^2\theta}\frac{\partial}{\partial\phi}\left(\underbrace{\frac{\partial}{\partial\phi}|\psi_{320}\rangle}_{0}\right)}\right]$$

ψ_{320} ϕ'ye bağlı değil

$$\langle L^2\rangle = -\hbar^2\langle\psi_{320}|\underbrace{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)|\psi_{320}\rangle}_{\bigstar}$$

$$\bigstar \Rightarrow \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)R_{32}(r)\,Y_2^0(\theta)$$

$$\Rightarrow R_{32}(r)\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\left(\frac{5}{16\pi}\right)^{1/2}(3\cos^2\theta-1)$$

$$\Rightarrow R_{32}(r)\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\left(\frac{5}{16\pi}\right)^{1/2}6\cos\theta(-\sin\theta)\right)$$

$$\Rightarrow R_{32}(r)\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\left(\frac{5}{16\pi}\right)^{1/2}(-6\cos\theta\sin^2\theta)\right]$$

$$\Rightarrow R_{32}(r)\frac{1}{\sin\theta}\left[\left(\frac{5}{16\pi}\right)^{1/2}\left(-6\left(-\sin^3\theta+\cos\theta\,2\sin\theta\cos\theta\right)\right)\right]$$

$$\Rightarrow R_{32}(r)\cdot\left(\frac{5}{16\pi}\right)^{1/2}\left(6\sin^2\theta-12\cos^2\theta\right) \qquad \sin^2\theta = 1-\cos^2\theta$$

$$\Rightarrow R_{32}(r)\cdot\left(\frac{5}{16\pi}\right)^{1/2}\left(6-6\cos^2\theta-12\cos^2\theta\right)$$

$$\Rightarrow R_{32}(r)\left(\frac{5}{16\pi}\right)^{1/2}6\left(1-3\cos^2\theta\right)$$

$$\Rightarrow -6\,\underbrace{R_{32}(r)\underbrace{\left(\frac{5}{16\pi}\right)^{1/2}(3\cos^2\theta-1)}_{Y_2^0(\theta)}}_{\psi_{320}} = -6\,\psi_{320}$$

$$\langle L^2\rangle = \langle\psi_{320}|L^2\rangle = -\hbar^2\langle\psi_{320}|\underbrace{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)|\psi_{320}\rangle}_{-6|\psi_{320}\rangle}$$

$$\langle L^2\rangle = 6\hbar^2\underbrace{\langle\psi_{320}|\psi_{320}\rangle}_{1} = 6\hbar^2$$

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