ANALYSIS II Homework 5 **26.04.2016**

1. Find the interval of convergence fo the series.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n$

(a) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2x+1)^{2n+1}}$

(b) $\sum_{n=1}^{\infty} \frac{1}{2n+1}\left(\frac{x-1}{x+1}\right)^{2n+1}$

(a) $\sum_{n=1}^{\infty} \frac{3n^3(x+1)^n}{5^n}$

(b) $\sum_{n=1}^{\infty} \frac{2n^3(2x-1)^n}{3^n}$

(c) $\sum_{n=1}^{\infty} \frac{n!(x-5)^n}{2^n}$

(d) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(e) $\sum_{n=1}^{\infty} (-1)^n n^2 x^n$

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^{2n}}{2n-1}$

(g) $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^4+16}$

(h) $\sum_{n=1}^{\infty} \frac{(-1)^n 10^n (x-10)^n}{n!}$

(ı) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n10^n}$

(i) $\sum_{n=1}^{\infty} \frac{n!(x-5)^n}{2^n}$

(j) $\sum_{n=1}^{\infty} \left(\frac{x^2+1}{5}\right)^n$

(k) $\sum_{n=1}^{\infty} \frac{(3-x)^n}{n^3}$

2. Find the Taylor expansion at the given point.

(a) $f(x)=\frac{2}{x-2},\ (x=1)$

(b) $f(x)=\frac{1}{x-1}+\frac{2}{x+1},\ (x=0)$

(c) $f(x)=\frac{1}{(x-1)(x+2)},\ (x=0)$

(d) $f(x)=e^{2x},\ (x=1)$

(e) $f(x)=\frac{1}{(2x-1)(x+1)},\ (x=0)$

(f) $f(x)=\sin 3x,\ (x=-\pi)$

(g) $f(x)=e^{2x-1},\ (x=-1)$

(h) $f(x)=e^{x+1},\ (x=2)$

(ı) $f(x)=\cos x,\ (x=\pi)$

(i) $f(x)=\sin x,\ \left(x=\frac{\pi}{2}\right)$

3. **(a)** Write the function $f(x)=x^4-4x^2+2x-1$ in pover of $(x-2)$.

(b) Write the function $f(x)=x^6-x^5+x^4-x^3+x^2-x+1$ in pover of $(x+1)$.

4. **(a)** Expand the function $f(x)=xe^{x}$ in power of $(x-1)$ for first four non-zero terms.

(b) Expand the function $f(x)=(x+1)e^{x+1}$ in power of $(x+1)$ for first 100 non-zero terms

(c) Expand the function $f(x)=e^{x}\cdot\sin x$ in power of $(x-\pi)$ for first five non-zero terms

(d) Expand the function $f(x)=\sqrt{x}$ in power of $(x-4)$ for first 1000 non-zero terms

(e) Expand the function $f(x)=\ln x$ in power of $(x-e)$ for first 10000 non-zero terms

5. Integrate the followings.

$$\int\left(ax^2+b\right)^3dx=?\quad \int\frac{\left(x^2+4\right)\left(x^2-2\right)dx}{\sqrt[3]{x^2}}=?\quad \int\frac{x^4+4}{x^2+2}dx=?\quad \int\frac{8xdx}{\sqrt{16-x^4}}dx=?$$

$$\int\sqrt{\frac{\arcsin x}{1-x^2}}dx=?\quad \int\frac{\arctan\left(x/2\right)}{4+x^2}dx=?\quad \int\frac{\cos(\ln x)\,dx}{x}=?\quad \int\frac{e^{x}dx}{e^{2}+e^{2x}}=?$$

$$\int\frac{x-\sqrt{\arctan(3x)}}{1+9x^2}dx=?\quad \int\frac{e^{x}dx}{\sqrt{e^{2}-e^{2x}}}=?\quad \int\frac{\pi dx}{x\ln^{\pi}x}=?\quad \int\frac{dx}{(x\ln x)(\ln(\ln x))}=?$$

$$\int\frac{dx}{\sqrt{x^2+4x}}=?\quad \int\frac{dx}{\sqrt{x^2-6x+10}}=?\quad \int\frac{dx}{\sqrt{-x^2+2x+3}}=?\quad \int\sqrt{x^2+4x}\,dx=?$$

$$\int\sqrt{x^2-6x+10}\,dx=?\quad \int\sqrt{-x^2+2x+3}\,dx=?\quad \int\frac{x\arcsin x dx}{\sqrt{1-x^2}}=?\quad \int\frac{2x}{x^2+2x+1}dx=?$$

$$\int\frac{2x-1}{x^2-2x+2}dx=?\quad \int\frac{8-x}{6x^2-5x-6}dx=?\quad \int\sqrt{x}\sin\sqrt{x}\,dx=?\quad \int\frac{xe^{x}dx}{\sqrt{1+e^{x}}}=?$$

$$\int\frac{\arcsin x dx}{\sqrt{1+x}}=?\quad \int\sin(3x+5)\cos(x-1)dx=?\quad \int\sin^4(3x)\cos^3(3x)dx=?$$

$$\int\frac{dx}{\cos x+2\sin x+5}=?\quad \int\frac{(\cos x+2\sin x)dx}{3\sin x+4\cos x+3}=?\quad \int\frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt[4]{x^5}+\sqrt[6]{x^7}}dx=?$$

$$\int\sqrt[3]{\frac{1-x}{1+x}}dx=?\quad \int\frac{\arcsin x dx}{x^2}=?\quad \int\frac{\arctan\left(e^{x}\right)dx}{e^{x}}=?\quad \int\arcsin\left(\frac{2\sqrt{x}}{1+x}\right)dx=?$$

$$\int\frac{dx}{\sqrt{1+x}+\sqrt[4]{1+x}}=?\quad \int\frac{dx}{\cos^2x+\sin^4x}=?\quad \int\frac{\cos 2x\,dx}{\cos^4x+\sin^4x}=?\quad \int\frac{x^3dx}{\sqrt{\left(1+2x^2\right)^3}}=?$$

$$\int\frac{x\,dx}{\sqrt{4-x^4}}=?\quad \int\frac{x^5\,dx}{\sqrt[3]{\sqrt{1+x^3}}}=?\quad \int\frac{\sqrt{x^3-4}\;dx}{x}=?\quad \int_{\pi/4}^{\pi/3}\frac{\ln(\tan x)}{\sin 2x}dx=?$$

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