1) $|\psi\rangle = \frac{1}{6}\left[\,|0\rangle + 0|1\rangle + 4|2\rangle\,\right]$ durumunda bir sistem ele alalım;

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & i \\ 0 & -i & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

operatörleri $|0\rangle,\ |1\rangle,\ |2\rangle$ bazında verilmiş olsun.

a) Let's carry out an experiment which measured A and B respectively. What is the probability 0 for A and 1 for B respectively.
b) Let's carry out an experiment which measured B and A respectively. What is the probability 0 for A and 1 for B respectively.
c) compare the results of a) and b).
d) Are {A}, { *B*} and {A, *B*} a complete set of commuting observable?

2) Let us consider a physical system, the Hilbert space of which can be defined by the span of three orthogonal states: $|u_1>$, $|u_2>$ and $|u_3>$. In the basis defined by these states (in the same order) we define two operators:

$$H = \hbar\omega\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad B = b\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

a) Can *H, B*, represent observable quantities?
b) Show that *H* and *B* commute. What is the most general form of the matrix that is commutable with *H.*
c) Find a basis of simultaneous eigenstates for *H* and *B.*
d) Are {*H, B*} a complete set of commuting operators? What about {H^2, *B*}

3) Suppose $\psi(x,0) = \frac{A}{x^2+a^2} \quad (-\infty<x<\infty)$ where A and a are constants.

(a) Determine A, by normalizing $\Psi(x, 0)$.
(b) Find $\langle x\rangle$, $\langle x^2\rangle$, and σ_x (at time $t = 0$).
(c) Find the momentum space wave function $\Phi(p, 0)$, and check that it is normalized.
(d) Use $\Phi(p, 0)$ to calculate $\langle p\rangle$, $\langle p^2\rangle$, and σ_p (at time $t = 0$).
(e) Check the Heisenberg uncertainty principle for this state.

4) The particle mass of m in the infinite quantum well (0<x<a). The initial state of the system is given by

$$\psi(x) = \frac{1}{\sqrt{10a}}\sin\left(\frac{\pi x}{a}\right) + A\sqrt{\frac{2}{a}}\sin\left(\frac{2\pi x}{a}\right) + \frac{3}{\sqrt{5a}}\sin\left(\frac{3\pi x}{a}\right)$$

a) Find A for normalized function.
b) What is the Energies and its probabilities.
c) Energy measured as $2\pi\hbar^2/ma^2$, what is the system state after this measurement.
d) Find $<x>$, $<p_x>$, $<X^2>$, $<(p_x)^2>$. Show that Heisenberg uncertainty principle ($\triangle p_x \triangle x$) by calculating $\triangle p_x$ and $\triangle x$.

5) Consider a three-dimensional vector space spanned by an orthonormal basis $|1>$, $|2>$, $|3>$. Kets $|\alpha>$ and $|\beta>$ are given by;

$$|\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle, \quad |\beta\rangle = i|1\rangle + 2|3\rangle.$$

(a) Construct $\langle\alpha|$ and $\langle\beta|$ (in terms of the dual basis $\langle 1|$, $\langle 2|$, $\langle 3|$).

(b) Find $\langle\alpha|\beta\rangle$ and $\langle\beta|\alpha\rangle$, and confirm that $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$.

(c) Find all nine matrix elements of the operator $\hat{A} \equiv |\alpha\rangle\langle\beta|$, in this basis, and construct the matrix **A**. Is it hermitian?

6) The Hamiltonian for a certain two-level system is

$$\hat{H} = \epsilon \left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right),$$

where $|1\rangle$, $|2\rangle$ is an orthonormal basis and ϵ is a number with the dimensions of energy. Find its eigenvalues and eigenvectors (as linear combinations of $|1\rangle$ and $|2\rangle$). What is the matrix **H** representing $\hat{H}$ with respect to this basis?

7)

(a) Show that the *sum* of two hermitian operators is hermitian.

(b) Suppose $\hat{Q}$ is hermitian, and α is a complex number. Under what condition (on α) is $\alpha\hat{Q}$ hermitian?

(c) When is the *product* of two hermitian operators hermitian?

(d) Show that the position operator ($\hat{x} = x$) and the hamiltonian operator ($\hat{H} = -(\hbar^2/2m)d^2/dx^2 + V(x)$) are hermitian.

8)

An operator A, corresponding to an observable α, has two normalised eigenfunctions ϕ_1 and ϕ_2, with eigenvalues a_1 and a_2. An operator B, corresponding to an observable β, has normalised eigenfunctions χ_1 and χ_2, with eigenvalues b_1 and b_2. The eigenfunctions are related by

$$\phi_1 = (2\chi_1 + 3\chi_2)/\sqrt{13}, \quad \phi_2 = (3\chi_1 - 2\chi_2)/\sqrt{13}.$$

α is measured and the value a_1 is obtained. If β is then measured and then α again, show that the probability of obtaining a_1 a second time is 97/169.