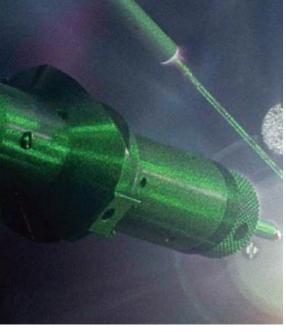
PHYSICS IV

COURSE TOPICS

- 1 The Birth of Modern Physics
- 2 Special Theory of Relativity
- 3 The Experimental Basis of Quantum Physics
- 4 Structure of the Atom
- 5 Wave Properties of Matter
- 6 Quantum Mechanics

Stephen T. Thornton MODERN for Scientists



principles of



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modern physics

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Fundamental Constants

Quantity	Symbol	Value(s)
Elementary charge	e	$1.6022 \times 10^{-19} \mathrm{C}$
Speed of light in vacuum	с	$2.9979 imes10^8~{ m m/s}$
Permeability of vacuum (magnetic constant)	μ_0	$4\pi imes 10^{-7} \mathrm{N} \cdot \mathrm{A}^{-2}$
Permittivity of vacuum (electric constant)	ϵ_0	$8.8542 imes 10^{-12} \mathrm{F \cdot m^{-1}}$
Gravitation constant	G	$6.6738 imes 10^{-11} \mathrm{N \cdot m^2 \cdot kg^{-2}}$
Planck constant	h	$6.6261 imes 10^{-34} { m J}\cdot{ m s}$
		$4.1357\times 10^{-15}~{\rm eV}\cdot{\rm s}$
Avogadro constant	$N_{ m A}$	$6.0221 \times 10^{23} \mathrm{~mol^{-1}}$
Boltzmann constant	k	$1.3807 imes 10^{-23} \mathrm{J} \cdot \mathrm{K}^{-1}$
Stefan-Boltzmann constant	σ	$5.6704 imes 10^{-8} \mathrm{W \cdot m^{-2} \cdot K^{-4}}$
Atomic mass unit	u	$1.66053886 imes 10^{-27} \text{ kg}$ 931.494061 MeV/ c^2

Particle Masses

		Mass in units of			
kg	MeV/c^2	u			
9.1094×10^{-31}	0.51100	5.4858×10^{-4}			
$1.8835 imes 10^{-28}$	105.66	0.11343			
$1.6726 imes 10^{-27}$	938.27	1.00728			
$1.6749 imes 10^{-27}$	939.57	1.00866			
$3.3436 imes 10^{-27}$	1875.61	2.01355			
$6.6447 imes 10^{-27}$	3727.38	4.00151			
	9.1094×10^{-31} 1.8835×10^{-28} 1.6726×10^{-27} 1.6749×10^{-27} 3.3436×10^{-27}	9.1094×10^{-31} 0.51100 1.8835×10^{-28} 105.66 1.6726×10^{-27} 938.27 1.6749×10^{-27} 939.57 3.3436×10^{-27} 1875.61			

Conversion Factors	
1 y = 3.156×10^7 s 1 lightyear = 9.461×10^{15} m 1 cal = 4.186 J 1 MeV/c = 5.344×10^{-22} kg · m/s 1 eV = 1.6022×10^{-19} J	1 T = 10^4 G 1 Ci = 3.7×10^{10} Bq 1 barn = 10^{-28} m ² 1 u = 1.66054×10^{-27} kg

Useful Combinations of Constants

$$\begin{split} &\hbar = h/2\pi = 1.0546 \times 10^{-34} \text{ J} \cdot \text{s} = 6.5821 \times 10^{-16} \text{ eV} \cdot \text{s} \\ &hc = 1.9864 \times 10^{-25} \text{ J} \cdot \text{m} = 1239.8 \text{ eV} \cdot \text{nm} \\ &\hbar c = 3.1615 \times 10^{-26} \text{ J} \cdot \text{m} = 197.33 \text{ eV} \cdot \text{nm} \\ &\frac{1}{4\pi\epsilon_0} = 8.9876 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \\ &\text{Compton wavelength } \lambda_c = \frac{h}{m_c c} = 2.4263 \times 10^{-12} \text{ m} \\ &\frac{e^2}{4\pi\epsilon_0} = 2.3071 \times 10^{-28} \text{ J} \cdot \text{m} = 1.4400 \times 10^{-9} \text{ eV} \cdot \text{m} \\ &\text{Fine structure constant } \alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = 0.0072974 \approx \frac{1}{137} \\ &\text{Bohr magneton } \mu_{\text{B}} = \frac{e\hbar}{2m_e} = 9.2740 \times 10^{-24} \text{ J/T} = 5.7884 \times 10^{-5} \text{ eV/T} \\ &\text{Nuclear magneton } \mu_{\text{B}} = \frac{e\hbar}{2m_e} = 5.0508 \times 10^{-27} \text{ J/T} \\ &= 3.1525 \times 10^{-8} \text{ eV/T} \\ &\text{Bohr radius } a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = 5.2918 \times 10^{-11} \text{ m} \\ &\text{Hydrogen ground state } E_0 = \frac{e^2}{8\pi\epsilon_0 a_0} = 13.606 \text{ eV} = 2.1799 \times 10^{-18} \text{ J} \\ &\text{Rydberg constant } R_{\infty} = \frac{\alpha^2 m_e c}{2h} = 1.09737 \times 10^7 \text{ m}^{-1} \\ &\text{Hydrogen Rydberg } R_{\text{H}} = \frac{\mu}{m_e} R_{\infty} = 1.09678 \times 10^7 \text{ m}^{-1} \\ &\text{Magnetic flux quantum } \Phi_0 = \frac{\hbar}{2e} = 2.0678 \times 10^{-15} \text{ T} \cdot \text{m}^2 \\ &\text{Classical electron radius } r_e = \alpha^2 a_0 = 2.8179 \times 10^{-15} \text{ m} \\ &kT = 2.5249 \times 10^{-2} \text{ eV} \approx \frac{1}{40} \text{ eV at } T = 293 \text{ K} \\ \end{split}$$

Note: The latest values of the fundamental constants can be found at the National Institute of Standards and Technology website at http://physics.nist.gov/cuu/Constants

The Birth of Modern Physics Lecture 1

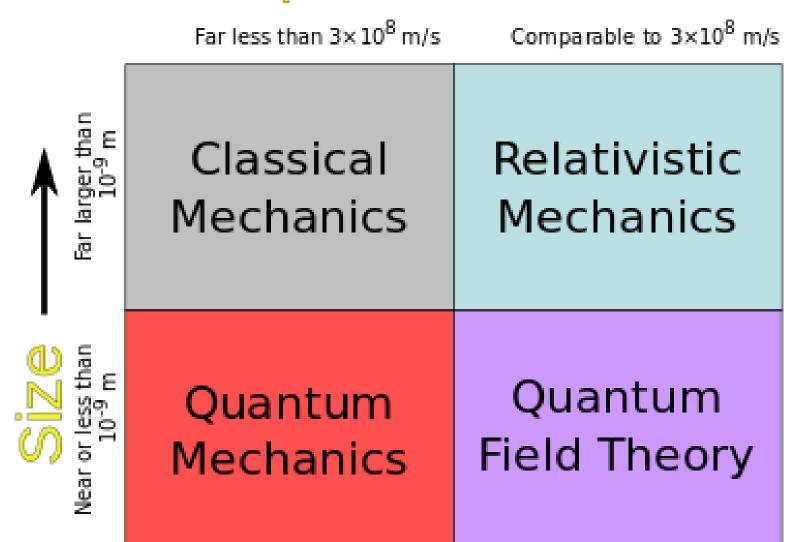
- The word **physics** is derived from the Latin word **physica**, which means **«natural thing»**.
- Physics is a branch of science that deals with the properties of matter and energy and the relationship between them.
- The scope of physics is very wide and vast. It deals with not only the tinniest particles of atoms, but also natural phenomenon like the galaxy, the milky way, solar and lunar eclipses, and more.

Branches of Physics

- Classical physics
- Modern physics -
- Nuclear physics
- Atomic physics
- Geophysics
- Biophysics
- Mechanics
- Acoustics
- Optics
- Thermodynamics
- Astrophysics

This branch is mainly concerned with the **theory of relativity** and **quantum mechanics.**





- The term modern physics generally refers to the study of those facts and theories developed in this century starting around 1900, that concern the ultimate structure and interactions of matter, space and time.
- The three main branches of classical physics such as mechanics, heat and electromagnetism – were developed over a period of approximately two centuries prior to 1900.
- Newton's mechanics dealt succesfully with the **motions of bodies** of macroscopic size moving with low speeds.
- This provided a foundation for many of the engineering accomplishments of the 18th and 19th centuries.

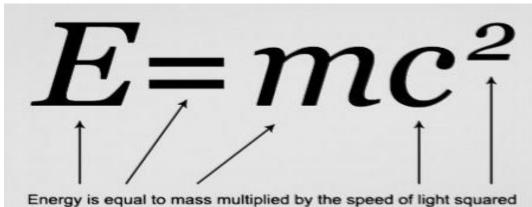
- Between 1837 1901, there were many remarkable achievements occured in physics.
- For example, the description and predictions of electromagnetism by **Maxwell**. Partly responsible for the rapid telecommunications of today.
- In this period, thermodynamics rose te become an exact science.
- Firstly in this lecture we will briefly revies the status of physics around 1895 which includes Newton's laws, Maxwell equations, the laws of thermodynamics.

The pioneers of modern physics

- Albert Einstein and Max Plank were the pioneers of modern physics as the first scientists to introduce the theory of relativity and quantum mechanics, respectively.
- In modern physics, energy and matter are not considered as separate entities. Rather, they are considered different forms of each other.
- The two pillars of modern physics are as follows :
- □ Albert Einstein's theory of relativity
- Max Plank's quantum theory

Theory of Relativity

- Albert Einstein's theory of relativity is one of the most important discoveries of the contemporary age, and states that the laws of physics are the same for all non-accelerating observers. As a result of this discovery, Einstein was able to confirm that space and time are interwoven in a single continuum known as space-time. As such, events that occur at the same time for one observer could occur at different times for another.
- Einstein's theory of relativity is summarized in the formula:



"E" represents energy "m" represents mass "c" represents the speed of light

Quantum Theory

- Discovered by Max Plank in 1900, quantum theory is the theoretical basis of modern physics that explains the nature and behavior of matter and energy on the atomic and subatomic level. The nature and behavior of matter and energy at that level is sometimes referred to as quantum physics and quantum mechanics.
- Plank discovered that **energy exists in individual units** in the same way that matter does, rather than just as a constant electromagnetic wave. Thus, **energy was quantifiable**. The existence of these units, called *quanta*, act as the basis of Plank's quantum theory.

Classical Physics of the 1890s

- The ideas of classical physics are just as important and useful today as they were at the end of the nineteenth century. The conservation laws of energy, linear momentum, angular momentum, and charge can be stated as follows:
- Conservation of energy: The total sum of energy (in all its forms) is conserved in all interactions.
- Conservation of linear momentum: In the absence of external forces, linear momentum is conserved in all interactions (vector relation).
- Conservation of angular momentum: In the absence of external torque, angular momentum is conserved in all interactions (vector relation).
- **Conservation of charge:** Electric charge is conserved in all interactions.

- A nineteenth-century scientist might have added the **conservation of mass** to this list, but we know it not to be valid today.
- These conservation laws are reflected in the laws of mechanics, electromagnetism, and thermodynamics.
- Electricity and magnetism, separate subjects for hundreds of years, were combined by James Clerk Maxwell (1831–1879) in his **four equations**.
- Maxwell showed **optics to be a special case of electromagnetism**. Waves, which permeated mechanics and optics, were known to be an important component of nature. Many natural phenomena could be explained by wave motion using the laws of physics.

Mechanics

- Important contributions to mechanics were made by astronomers because of the great interest in the heavenly bodies.
- Galileo (1564–1642), the first great experimenter.
- Newton (1642–1727), the greatest scientist of his time.
- We owe to Newton our present understanding of motion. He understood clearly the relationships among position, displacement, velocity, and acceleration. He understood how motion was possible and that a body at rest was just a special case of a body having constant velocity.
- Newton pointed out that the motions of the planets about our sun can be understood by the same laws that explain motion on Earth, like apples falling from trees.

- Newton was able to elucidate the relationship **between net force and acceleration**.
- His concepts were stated in three laws that bear his name today: Newton's Law

Newton's first law: An object in motion with a **constant velocity** will **continue in motion** unless acted upon by some net external force. A body at rest is just **a special case of** Newton's first law with **zero velocity.** Newton's first law is often called the *law of inertia* and is also used to describe inertial reference frames.

Newton's second law: The acceleration *a* of a body is proportional to the net external force *F* and inversely proportional to the mass *m* of the body.

It is stated mathematically as

$$\vec{F} = m\vec{a}$$

• A more general statement relates force to the time rate of change of the linear momentum \vec{p} .

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Newton's third law: The force exerted by body 1 on body 2 is **equal in magnitude and opposite in direction** to the force that body 2 exerts on body 1. If the force on body 2 by body 1 is denoted by $\overline{F_{21}}$, then Newton's third law is written as

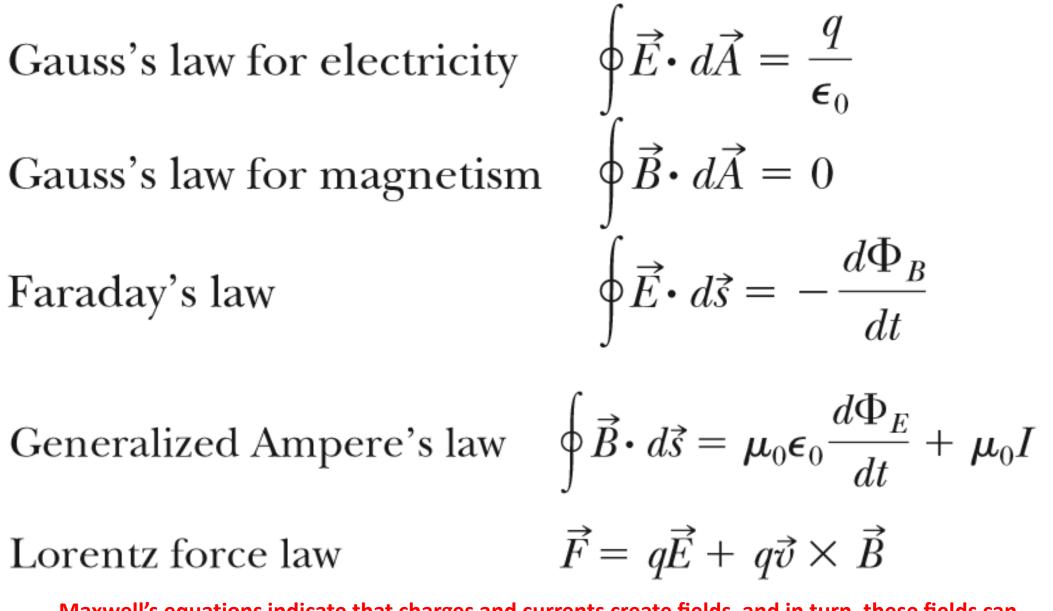
$$\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$$

It is often called the law of action and reaction.

- These three laws **develop the concept of force**.
- Using that concept together with the concepts of velocity *v*, acceleration *a*, linear momentum *p*, rotation (angular velocity *v* and angular acceleration *a*), and angular momentum *L*, we can describe the complex motion of bodies.

Electromagnetism

- Important contributions were made by Charles Coulomb (1736– 1806), Hans Christian Oersted (1777–1851), Thomas Young (1773– 1829), André Ampère (1775–1 836), Michael Faraday (1791–1867), Joseph Henry (1797–1 878), James Clerk Maxwell (1831–1 879), and Heinrich Hertz (1857–1 894).
- Maxwell showed that **electricity and magnetism were intimately connected** and were related by a change in the inertial frame of reference.
- Maxwell's four equations, together with the Lorentz force law, explain much of electromagnetism.



Maxwell's equations indicate that charges and currents create fields, and in turn, these fields can create other fields, both electric and magnetic.

Thermodynamics

- Thermodynamics deals with temperature T, heat Q, work W, and the internal energy of systems U.
- The understanding of the concepts used in thermodynamics— such as pressure P, volume V, temperature, thermal equilibrium, heat, entropy, and especially energy.
- The concepts of pressure and volume as mechanical properties.
- We have learned that the internal energy of a system of noninteracting point masses depends only on the temperature.

• The primary results of thermodynamics can be described in two laws:

First law of thermodynamics: The change in the internal energy ΔU of a system is equal to the heat Q added to the system plus the work W done on the system.

 $\Delta \boldsymbol{U} = \boldsymbol{Q} + \boldsymbol{W}$

The first law of thermodynamics generalizes the conservation of energy by including heat.

Second law of thermodynamics: It is not possible to convert heat completely into work without some other change taking place. The second law says what kinds of energy processes cannot take place.

For example, it is possible to completely **convert work into heat, but not vice versa**, without some other change taking place.

- One of the other laws of thermodynamics is called «zeroth law».
- It states that if two thermal systems are in thermodynamic equilibrium with a third system, they are in equilibrium with each other. We can state it more simply by saying that two systems at the same temperature as a third system have the same temperature as each other. This concept was not explicitly stated until the twentieth century.
- The "third" law of thermodynamics expresses that it is **not possible to** achieve an absolute zero temperature.

The Kinetic Theory of Gases

- Experiments were relatively easy to perform on gases.
- Around 1662 Irish chemist Boyle showed that the pressure times the volume of a gas was constant for a constant temperature.
- The relation PV = constant (for constant T) is now referred to as **Boyle's law**.
- French physicist Jacques Charles found that V/T = constant (at constant pressure), referred to as Charles's law.
- We obtain the **ideal gas equation**:

$$PV = nRT$$

where n is the number of moles and R is the ideal gas constant, 8.31 J/mol.K

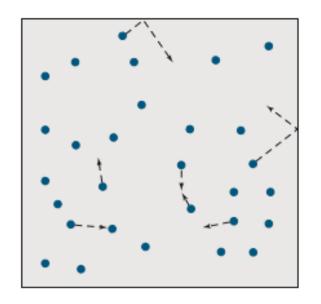


Figure 1.1 Molecules inside a closed container are shown colliding with the walls and with each other. The motions of a few molecules are indicated by the arrows. The number of molecules inside the container is huge.

- The kinetic theory of gases is usually taught by applying Newton's laws to the collisions that a molecule makes with other molecules and with the walls.
- A representation of a few molecules colliding is shown in Figure.
- In the simple model of an ideal gas, only elastic collisions are considered.
- By taking averages over the collisions of many molecules, the ideal gas law, is revealed.
- The average kinetic energy of the molecules is shown to be linearly proportional to the temperature, and the internal energy U is

n :the number of moles of gas,
N_A : Avogadro's number,
<K> : the average kinetic energy of a molecule
R : the ideal gas constant.

$$U = n N_{\rm A} \langle K \rangle = rac{3}{2} n R T$$
 Statistical thermodynamics

- This relation ignores any nontranslational contributions to the molecular energy, such as **rotations and vibrations**.
- However, energy is not represented only by translational motion. It became clear that all degrees of freedom, including **rotational and vibrational**, were also capable of carrying energy.
- The equipartition theorem states that each degree of freedom of a molecule has an average energy of kT/2, where k is the Boltzmann constant (k = R/N_A).
- Translational motion has three degrees of freedom, and rotational and vibrational modes can also be excited at higher temperatures.
- If there are **f degrees of freedom**, then equation becomes

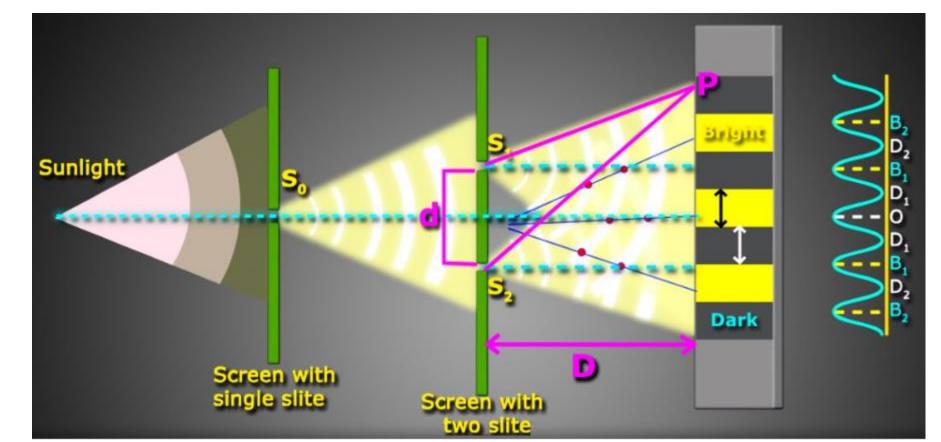
$$U = \frac{f}{2} nRT \qquad \text{Internal energy}$$

Waves and Particles

- We first learned the concepts of velocity, acceleration, force, momentum, and energy in introductory physics by using a single particle with its mass concentrated in one small point.
- In order to adequately describe nature, we add two- and threedimensional bodies and rotations and vibrations.
- The kinetic energy of a moving particle is one way that energy can be transported from one place to another.
- We have found that many natural phenomena can be explained only in terms of **waves**, which are traveling disturbances that carry energy.
- This description includes standing waves, which are superpositions of traveling waves.

- Most waves, like water waves and sound waves, need an elastic medium in which to move.
- Matter is not transported in waves—but energy is.
- Mass may oscillate, but it doesn't actually propagate along with the wave.
- Example; The boat also reacts to the wave, but it primarily rocks back and forth, throwing around things that are not fixed on the boat. The **boat obtains considerable kinetic energy from the wave**. After the wave passes, the boat eventually **returns to rest.**

- After many discussions on **behaviour of light**, in 1802 Thomas Young annunced the results of his two-slit interference experiment.
- This experiment indicated that light behaved as a wave.
- Fresnel showed the same results.
- By 1830 most physicists believed in the wave theory.



Young's Double Slit Experiment

- The bright and dark regions can be understood only if light is a wave and not a particle.
- In the 1860s Maxwell showed that electromagnetic waves consist of oscillating electric and magnetic fields. Visible light covers just a narrow range of the total electromagnetic spectrum, and all electromagnetic radiation travels at the speed of light c in free space, given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \lambda f$$

- λ is the wavelength and f is the frequency. The fundamental constants μ_0 and ϵ_0 are defined in electricity and magnetism and reveal the connection to the speed of light.
- In 1887 the German physicist Heinrich Hertz (1857–1 894) succeeded in generating and detecting electromagnetic waves having wavelengths far outside the visible range ($\lambda \cong 5$ m).

Conservation Laws and Fundamental Forces

- Conservation laws are the guiding principles of physics.
- They include energy, linear momentum, angular momentum, and charge.
- We will study the Newtonian or Galilean invariance and find it lacking in our study of relativity; a new invariance principle will be needed.

Fundamental Forces

- There are only three fundamental forces.
- All the other forces can be derived from them.
- These three forces are the **gravitational**, **electroweak**, and **strong** forces.
- The approximate strengths and ranges of the three fundamental forces are listed in Table.

Interaction		Relative Strength *	Range
Strong		1	Short, $\sim 10^{-15}$ m
Electroweak	Electromagnetic	10^{-2}	Long, $1/r^2$
	∫ _{Weak}	10^{-9}	Short, $\sim 10^{-15}$ m
Gravitational		10^{-39}	Long, $1/r^2$

*These strengths are quoted for neutrons and/or protons in close proximity.

- Physicists sometimes use the term interaction when referring to the fundamental forces.
- The **gravitational force** is the weakest. It is the force of mutual attraction between masses and, according to Newton, is given by

$$\vec{F}_g = -G\frac{m_1m_2}{r^2}\,\hat{r}$$

where m_1 and m_2 are two point masses, G is the gravitational constant, r is the distance between the masses, and \hat{r} is a unit vector directed along the line between the two point masses (attractive force).

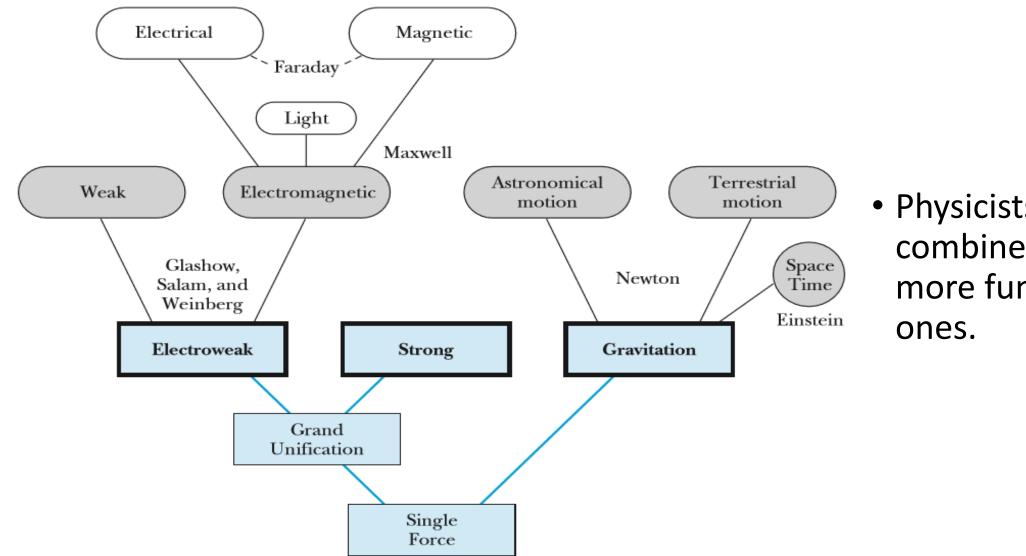
- The gravitational force is noticeably effective only on a macroscopic scale.
- Gravity is a long-range force that diminishes as $1/r^2$.

- In the 1970s Sheldon Glashow, Steven Weinberg, and Abdus Salam predicted that the electromagnetic and weak forces were in fact facets of the <u>same force</u>.
- For all practical purposes, **the weak interaction is effective in the nucleus** only over distances the size of 10⁻¹⁵ m. Except when dealing with very high energies, physicists mostly treat nature as if the electromagnetic and weak forces were separate. Therefore, you will sometimes see references to the four fundamental forces (gravity, strong, electromagnetic, and weak).
- Electromagnetic force is responsible for practically all nongravitational forces that we experience. The electrostatic, or Coulomb, force between two point charges q₁ and q₂, separated by a distance r, is given by

$$\vec{F}_C = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

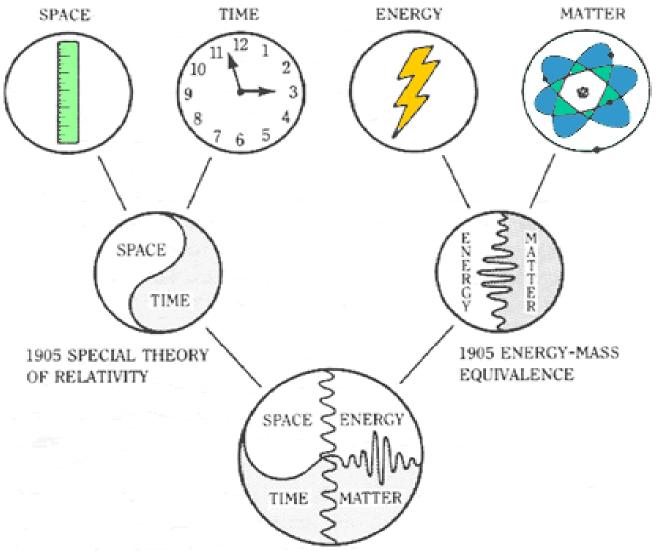
- The easiest way to remember the vector direction is that **like charges repel and unlike charges attract.** Moving charges also create and react to magnetic fields.
- The third fundamental force, the strong force, is the one holding the nucleus together. It is the strongest of all the forces.
- The strong force is so strong that it easily binds two protons inside a nucleus even though the electrical force of repulsion over the tiny confined space is huge.

Unification of forces



 Physicists strive to combine forces into more fundamental ones.

Relativity

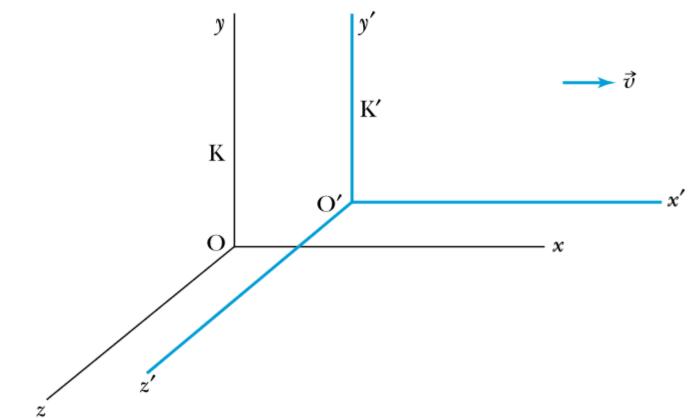


1915 GENERAL THEORY OF RELATIVITY

Special Theory of Relativity

- One of the great theories of physics appeared early in the twentieth century when Albert Einstein presented his special theory of relativity in 1905.
- Newton's laws of motion must be measured relative to some reference frame.
- A reference frame is called an **inertial frame** if Newton's laws are valid in that frame.
- If a body subject to no net external force moves in a straight line with constant velocity, then the coordinate system attached to that body defines an inertial frame.
- If Newton's laws are valid in one reference frame, then they are also valid in a reference frame moving at a uniform velocity relative to the first system.
- This is known as the **Newtonian principle of relativity** or **Galilean invariance**.

- Newton showed that it was not possible to determine absolute motion in space by any experiment, so **he decided to use relative motion**.
- The Newtonian concepts of time and space are completely separable.
- Consider two inertial reference frames, K and K', that move along their x and x' axes, respectively, with uniform relative velocity v as shown in Figure.

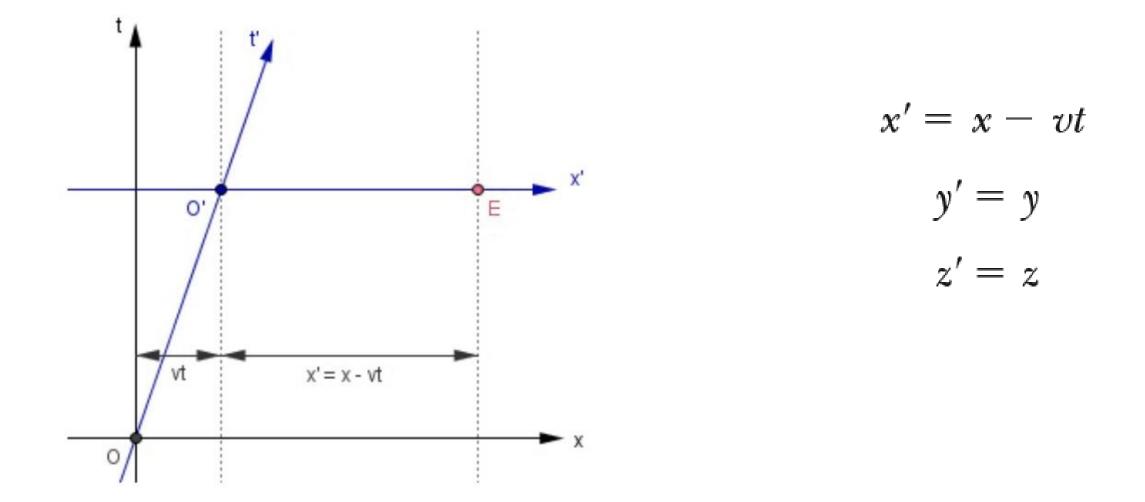


Two inertial systems are moving with relative speed v along their x axes. We show the system K at rest and the system K' moving with speed v relative to the system K.

- System K' moving to the right with velocity v with respect to system K, which is fixed or stationary somewhere.
- One result of the relativity theory is that there are no fixed, absolute frames of reference.
- We use the term fixed to refer to a system that is fixed on a particular object, such as a planet, star, or spaceship that itself is moving in space.
- The transformation of the coordinates of a point in one system to the other system is given by

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$

Graphically:



Source: Susskind Lectures, Special Relativity, Lecture 1 (Galilean relativity).

• Similarly, the inverse transformation is given by

$$x = x' + vt$$
$$y = y'$$
$$z = z'$$

where we have set t = t' because Newton considered time to be absolute.

• These equations are known as Galilean transformation.

- Newton's laws of motion are invariant under a Galilean transformation; that is, they have the same form in both systems K and K'.
- Newton's laws of motion had the same form under a Galilean transformation.
- But Maxwell equations did not. And Einstein believed in Maxwell's equations and he believed there is a problem about Newtonian principle of relativity.
- In 1905 he proposed that space and time are not separate and that Newton's laws are only an approximation.
- First we have to see the experimental situation historically and the problem behind.

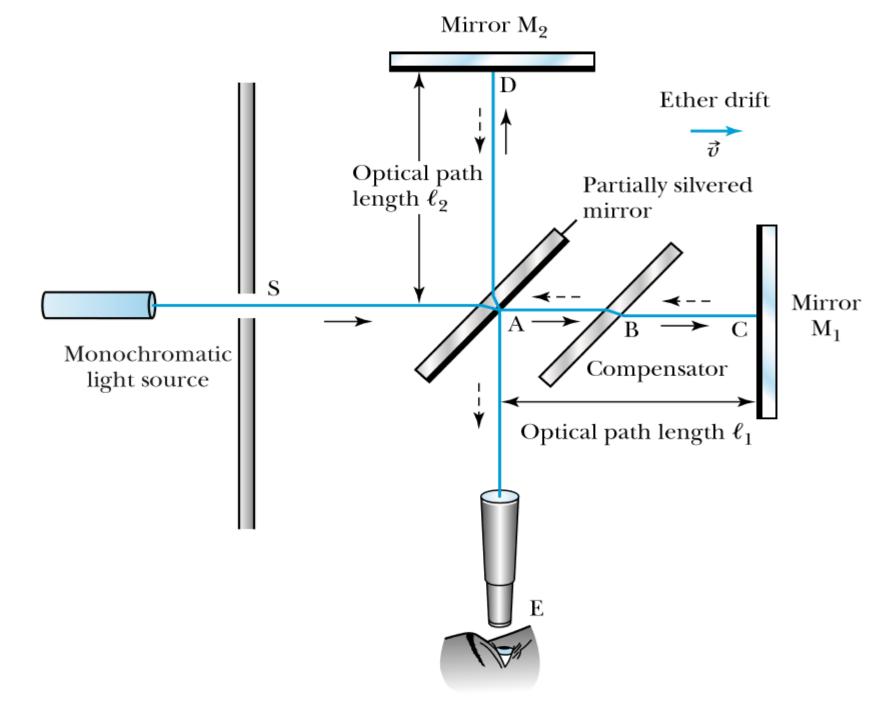
The Apparent Need for Ether

- Thomas Young, an English physicist and physician, performed his famous experiments on the interference of light in 1802.
- A decade later, the French physicist and engineer Augustin Fresnel published his calculations showing the detailed understanding of <u>interference, diffraction, and polarization</u>.
- All known waves (other than light) require a medium in which to propagate (water waves have water, sound waves have, for example, air, and so on).
- Light also required a medium, even though light was apparently able to travel in vacuum through outer space.
- This medium was called the **luminiferous ether** or just **ether** for short.

- The ether had to have such a low density that planets could pass through it, seemingly for eternity, with no apparent loss of orbit position. Its elasticity must be strong enough to pass waves of incredibly high speeds!
- The concept of ether was well accepted by 1880.
- Maxwell's equations predict the velocity of light in vacuum to be c.
- If we have a flashbulb go off in the moving system K', an observer in system K' measures the speed of the light pulse to be c.
- With Galilean transformations we find the speed measured in system K to be c + v.
- Physicists of the late nineteenth century proposed that there must be one preferred inertial reference frame in which the ether was stationary and that in this system the speed of light was c.
- Scientists set out to find the effects of the ether.

The Michelson-Morley Experiment

- The Earth orbits around the sun at a high orbital speed, about 10⁻⁴ c, so an obvious experiment is to try to find the effects of the Earth's motion through the ether.
- Michelson in 1800s built an extremely precise device called an interferometer, which measures the phase difference between two light waves.
- Michelson used his interferometer to detect the difference in the speed of light passing through the ether in different directions.

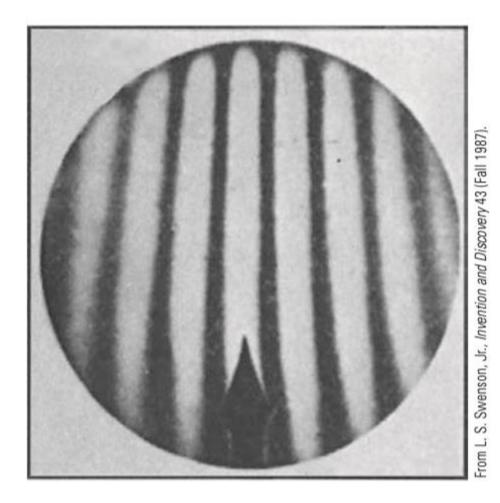


A schematic diagram of Michelson's interferometer experiment.

Light of a single wavelength is partially reflected and partially transmitted by the glass at A.

The light is subsequently reflected by mirrors at C and D, and, after reflection or transmission again at A, enters the telescope at E.

Interference fringes are visible to the observer at E.



The observed interference pattern consists of alternating bright and dark bands, corresponding to constructive and destructive interference, respectively.

For constructive interference, the difference between the two path lengths (to and from the mirrors) is given by some number of wavelengths, $2(l_1 - l_2) = n\lambda$ where l is the wavelength of the light and n is an integer.

- The expected shift in the interference pattern can be calculated by determining the time difference between the two paths.
- The total time for the round-trip journey to mirror M_1 is t_1 :

$$t_1 = \frac{\ell_1}{c+v} + \frac{\ell_1}{c-v} = \frac{2c\ell_1}{c^2 - v^2} = \frac{2\ell_1}{c} \left(\frac{1}{1 - v^2/c^2}\right)$$

• The time t₂ for the light to pass to mirror M₂ at D and back is

$$t_2 = \frac{2\ell_2}{\sqrt{c^2 - v^2}} = \frac{2\ell_2}{c} \frac{1}{\sqrt{1 - v^2/c^2}}$$

• The time difference between the two journeys Δt is

$$\Delta t = t_2 - t_1 = \frac{2}{c} \left(\frac{\ell_2}{\sqrt{1 - v^2/c^2}} - \frac{\ell_1}{1 - v^2/c^2} \right)$$

- We now rotate the apparatus by 90° so that the ether passes along the length l_2 toward the mirror M_2 .
- The time difference $\Delta t'$ is now

$$\Delta t' = t'_2 - t'_1 = \frac{2}{c} \left(\frac{\ell_2}{1 - v^2/c^2} - \frac{\ell_1}{\sqrt{1 - v^2/c^2}} \right)$$

• The time difference is

$$\Delta t' - \Delta t = \frac{2}{c} \left(\frac{\ell_1 + \ell_2}{1 - v^2/c^2} - \frac{\ell_1 + \ell_2}{\sqrt{1 - v^2/c^2}} \right)$$

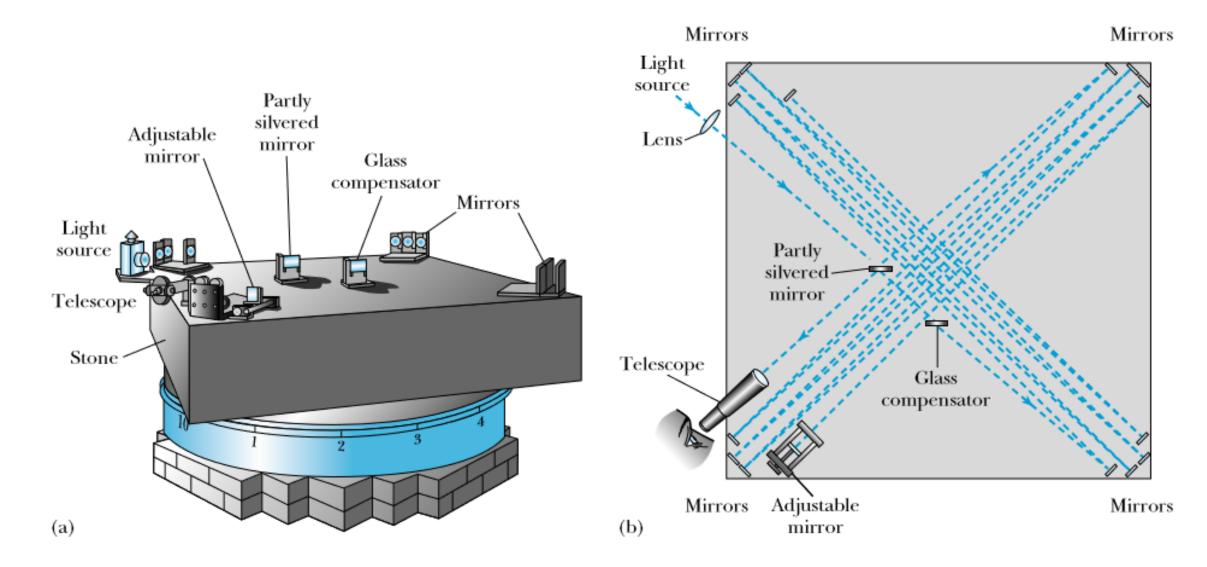
• c >> v, we can use the binomial expansion to expand the terms involving $\frac{v^2}{c^2}$, keeping only the lowest terms.

$$\Delta t' - \Delta t = \frac{2}{c} (\ell_1 + \ell_2) \left[\left(1 + \frac{v^2}{c^2} + \cdots \right) - \left(1 + \frac{v^2}{2c^2} + \cdots \right) \right]$$
$$\approx \frac{v^2 (\ell_1 + \ell_2)}{c^3}$$

when $l_1 = l_2 = l = 1.2 m$ thus Equation predicts a time difference of $8 x 10^{-17} s$.

- This time difference represents 0.04 fringes in the interference pattern.
- Michelson reasoned that he should be able to detect a shift of at least half this value but found none.

Michelson-Morley performed more sophisticated experiment in 1887.



- The new experiment had an optical path length of 11 m, created by reflecting the light for eight round trips.
- The new apparatus was mounted on soapstone that floated on mercury to eliminate vibrations
- And so effective that Michelson and Morley believed they could detect a fraction of a fringe shift as small as 0.005.
- They reported in 1887 a null result—no effect!
- The ether does not seem to exist.
- There seems to be no single reference inertial system in which the speed of light is actually c.