The Experimental Basis of Quantum Physics

X-Ray Production

- In the **photoelectric effect**, a **photon** gives up all of its energy to an **electron**, which may then **escape from the material** in which it was bound.
- Can the **inverse process** occur? Can an electron (or any charged particle) give up its energy and create a photon?
- **The answer is <u>yes</u>, but the process must be consistent with the laws of physics**.
- **Recall that photons must be created or absorbed as whole units**.
- A photon **cannot give up half** its energy; it must give up all its energy.
- If in some physical process only part of the photon's energy were required, then a ***new*** photon would be created to carry away the remaining energy.

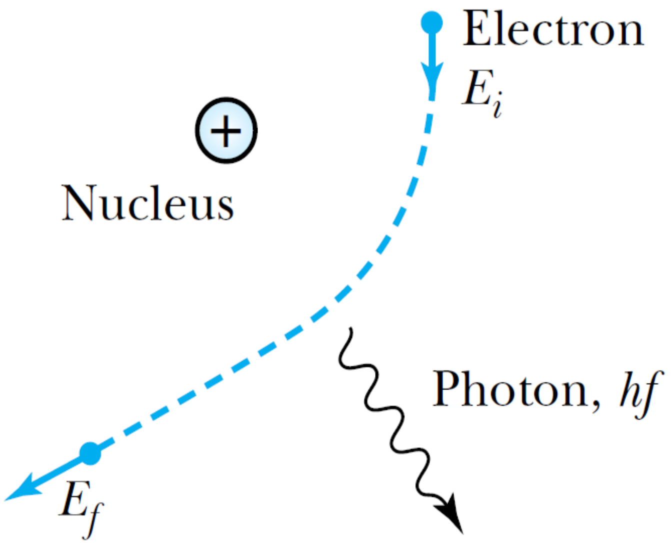
- Unlike a photon, an **electron** may give up **part or all of its kinetic energy** and still **be the same electron**.
- When an **electron interacts with the strong electric field** of the atomic nucleus and is consequently accelerated, the electron radiates electromagnetic energy.
- According to classical electromagnetic theory, it should do so continuously.
- In the quantum picture we must think of the **electron as emitting a series of photons with varying energies**; this is the only way that the **inverse photoelectric effect can occur**.
- An energetic electron passing through matter will radiate photons and lose kinetic energy.
- The process by which photons are emitted by an electron slowing down is called **bremsstrahlung**, from the German word for "**braking radiation**."

X-Ray Production

- As shown in the figure, an electron of energy E_i passing through the electric field of a nucleus is accelerated and produces a photon of energy $E = hf$.
- The final energy of the electron is determined from the conservation of energy to be

$$E_f = E_i - hf$$

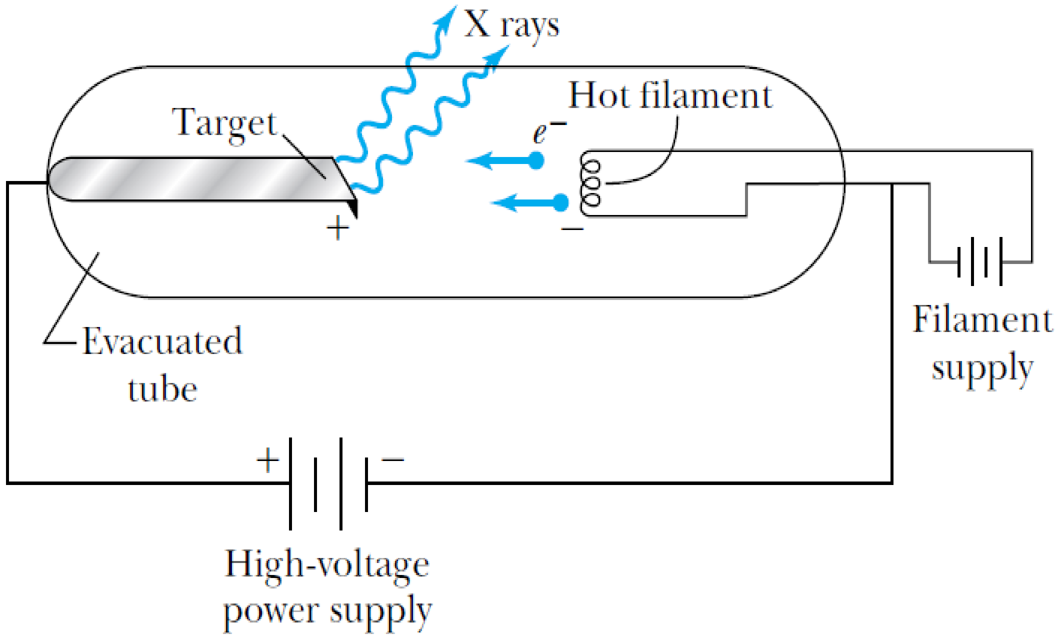
- Because **linear momentum must be conserved**, the nucleus absorbs very little energy, and it is ignored.
- One or more photons may be created in this way as **electrons pass through matter**.



Bremsstrahlung is a process through which an electron is accelerated while under the influence of the nucleus.
The accelerated electron emits a photon.

X-Ray Production

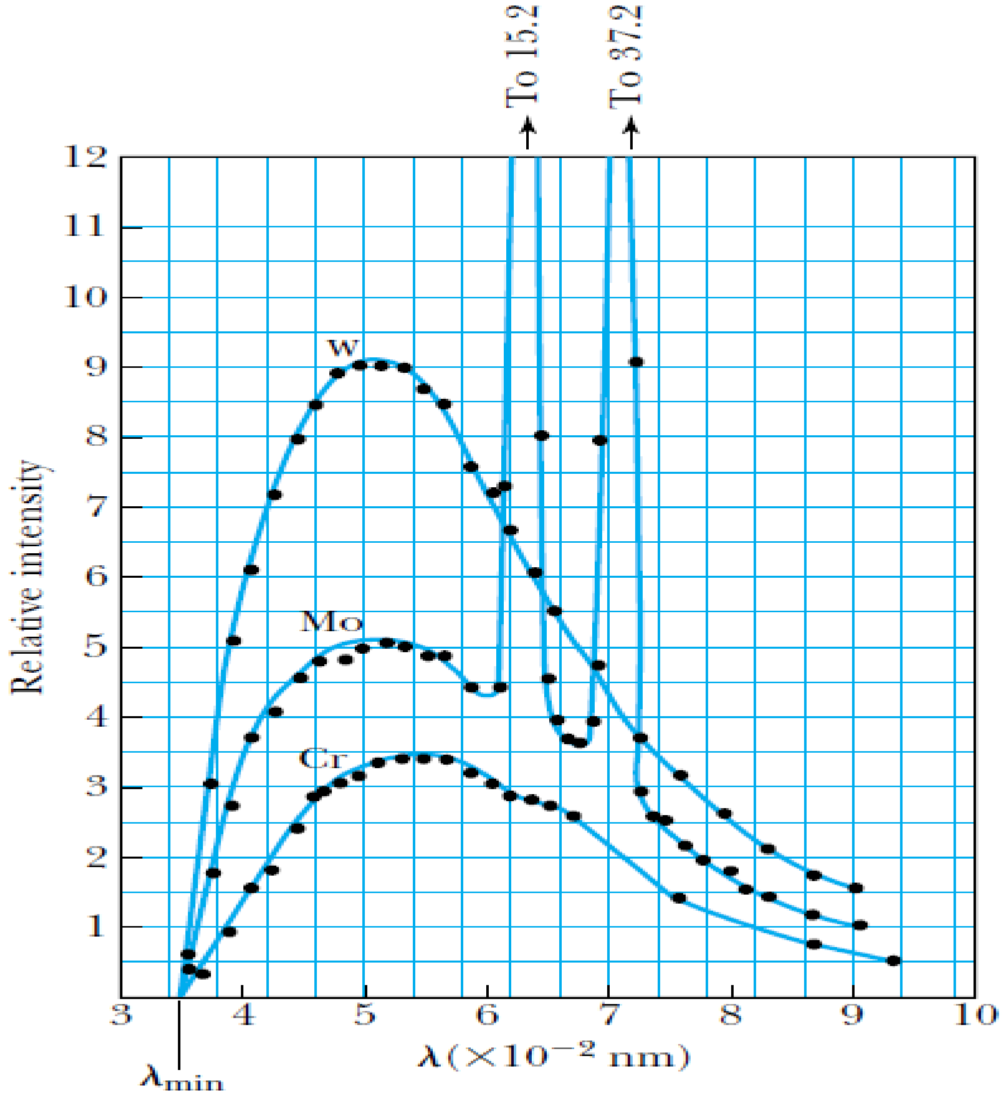
- The X-rays are produced by the **bremsstrahlung effect** in an apparatus shown in Figure.
- **Current** passing through a filament produces copious numbers of **electrons by thermionic emission**.
- These **electrons** are focused by the cathode structure into a beam and are **accelerated by potential differences of thousands of volts** until they impinge on a metal anode surface, producing **X-rays** by bremsstrahlung (and other processes) as they stop in the anode material.
- Much of the **electron's kinetic energy is lost** by heating the anode material and not by bremsstrahlung.
- The x-ray tube is evacuated so that the air between the filament and anode will not scatter the electrons.



- The X-rays produced pass through the sides of the tube and can be used for a large number of applications, including **medical diagnosis and therapy, fundamental research in crystal and liquid structure, and engineering diagnoses of flaws in large welds and castings**.

X-Ray Production

- **X-rays** from a standard tube include **photons of many wavelengths**.
- By scattering **X-rays from crystals** we can produce strongly collimated **monochromatic (single-wavelength) x-ray beams**.
- Early x-ray spectra produced by x-ray tubes of accelerating **potential 35 kV** are shown in Figure.
- These particular tubes had targets of **tungsten, molybdenum, and chromium**.
- The smooth, **continuous x-ray spectra** are those produced by bremsstrahlung, and the sharp "**characteristic x rays**" are produced by atomic excitations.
- X-ray wavelengths typically range from **0.01 to 1 nm**.
- However, high-energy accelerators can produce x rays with wavelengths as short as **10^{-6} nm**.



The relative intensity of X-rays produced in an X-ray tube is shown for an accelerating voltage of 35 kV.
Notice that **λ_{min} is the same for all three targets**.
From C. T. Ulrey, Physical Review ***11****, 405 (1918).*

X-Ray Production

- Notice that in Figure, the **minimum wavelength** λ_{min} for all **three targets is the same**.
- The **minimum wavelength** λ_{min} corresponds to the **maximum frequency** f_{max}.
- If the electrons are accelerated through a voltage V_0, then their kinetic energy is $\boldsymbol{eV_0}$.
- The maximum photon energy therefore occurs when the electron gives up all of its kinetic energy and creates one photon (this is relatively unlikely, however).
- This process is the **inverse photoelectric effect**.
- The **conservation of energy** requires that the **electron kinetic energy** equal the **maximum photon energy** (where we neglect the work function ϕ because it is normally so small compared with eV_0).

$$\mathrm{e}V_0 = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{e}\frac{1}{V_0} = \frac{1{,}240 \times 10^{-6}\ V \cdot m}{V_0}$$

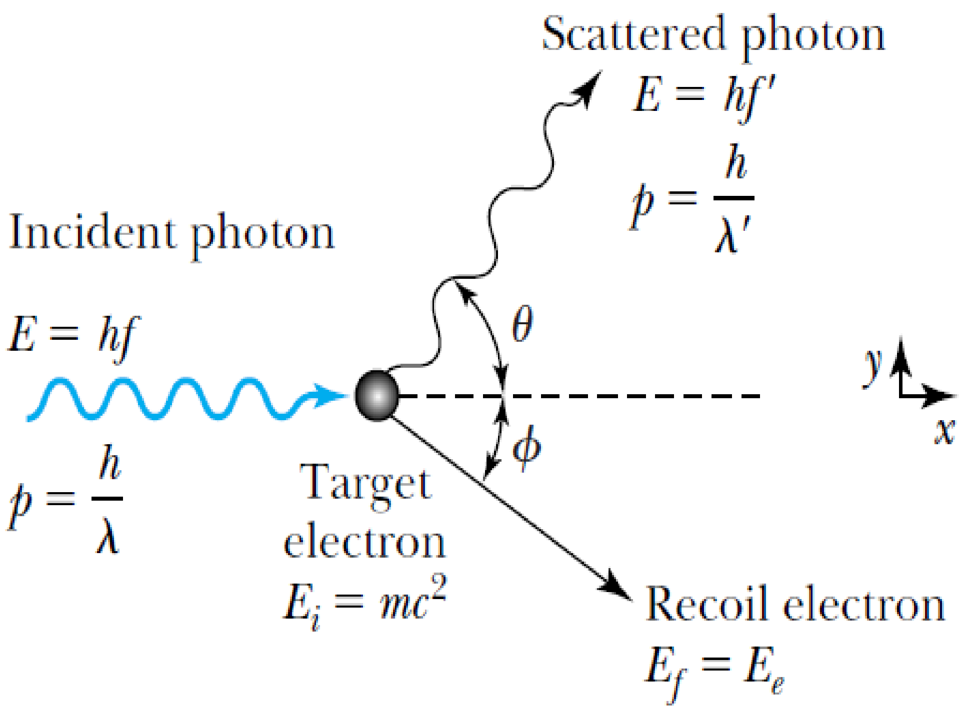
Duane-Hunt rule

X-Ray Production

- The relation was first found experimentally and is known as the **Duane-Hunt rule** (or limit).
- **Its explanation in 1915 by the quantum theory** is now considered further **evidence of Einstein's photon concept.**
- The **value λ_{min} depends only on the accelerating voltage and is the same for all targets.**
- **Only the quantum hypothesis explains all of these data**.
- Because the heavier elements have stronger nuclear electric fields, they are more effective in accelerating electrons and making them radiate.
- The **intensity of the X-rays increases with the square** of the atomic number of the target.
- The **intensity** is also approximately **proportional to the square of the voltage** used to accelerate the electrons.
- This is why high voltages and tungsten anodes are so often used in x-ray machines.
- **Tungsten also has a very high melting temperature and can withstand high electron-beam currents.**

Compton Effect

- When a **photon enters matter**, it is likely to interact with **one of the atomic electrons**.
- According to **classical theory**, the **electrons** will oscillate at the photon frequency because of the interaction of the electron with the electric and magnetic field of the photon and will reradiate electromagnetic radiation (photons) at this same frequency.
- **This is called Thomson scattering**.
- However, in the early 1920s Arthur Compton experimentally confirmed an earlier observation by J. A. Gray that, especially at **backward-scattering angles**, there appeared to be a component of the emitted radiation (called a modified wave) that had a **longer wavelength than the original primary** (unmodified) wave.
- Classical electromagnetic theory **cannot explain this** modified wave.
- Compton then attempted to understand theoretically such a process and could find only one explanation: ***Einstein's photon particle concept must be correct.***



Compton scattering of a photon by an electron essentially at rest.

Compton Effect

- Compton proposed in 1923 that the **photon is scattered from only one electron**, **rather than from all the electrons in the material**, and that the laws of the conservation of energy and momentum apply as in any **elastic collision between two particles**.
- We recall that the **momentum of a particle moving at the speed of light** (photon) is given by

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

- We treat the **photon as a particle with a definite energy and momentum**.
- Scattering takes place in a plane, which we take to be the *xy* plane in Figure.
- Both ***x* and *y* components of momentum must be conserved**, because of the vector nature of the linear momentum.
- The energy and momentum before and after the collision (treated relativistically) are given in Table.
- The incident and scattered photons have frequencies f and f', respectively.
- The recoil electron has energy E_e and momentum p_e .

Compton Effect

- In the **final system the electron's total energy** is related to its momentum by Equation :

$$E_e^2 = (mc^2)^2 + p_e^2 c^2$$

- We can write the conservation laws now, **initial = final,** as

$$Energy \quad hf + mc^2 = hf' + E_e$$

$$p_x \qquad \frac{h}{\lambda} = \frac{h}{\lambda'} \cos\theta + p_e \cos\phi$$

$$p_y \qquad \frac{h}{\lambda'} \sin\theta = p_e \sin\phi$$

Compton Effect

- We will relate the change in wavelength Δλ = λ′ - λ to the scattering angle θ of the photon.
- We first **eliminate the recoil angle ϕ by squaring Equations and adding them**, resulting in

$$p_e^2 = \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\left(\frac{h}{\lambda}\right)\left(\frac{h}{\lambda'}\right)\cos\theta$$

Table 3.4 Results of Compton Scattering

Energy or Momentum	Initial System	Final System
Photon energy	hf	hf'
Photon momentum in x direction (p_x)	$\frac{h}{\lambda}$	$\frac{h}{\lambda'}\cos\theta$
Photon momentum in y direction (p_y)	0	$\frac{h}{\lambda'}\sin\theta$
Electron energy	mc^2	$E_e = mc^2 + \text{K.E.}$
Electron momentum in x direction (p_x)	0	$p_e\cos\phi$
Electron momentum in y direction (p_y)	0	$-p_e\sin\phi$

Compton Effect

- Then we substitute $\boldsymbol{E_e}$ from one Equation and p_e from other Equation into the third Equation (setting λ = *c/f*).

$$[h(f - f' + mc^2)]^2 = m^2c^4 + (hf)^2 + (hf')^2 - 2(hf)(hf')\cos\theta$$

- Squaring the left-hand side and canceling terms leaves

$$mc^2(f - f') = hff'(1 - \cos\theta)$$

- Rearranging terms gives

$$\frac{h}{mc^2}(1 - \cos\theta) = \frac{f - f'}{ff'} = \frac{\frac{c}{\lambda} - \frac{c}{\lambda'}}{\frac{c^2}{\lambda\lambda'}} = \frac{1}{c}(\lambda' - \lambda)$$

$$\boldsymbol{\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)}$$

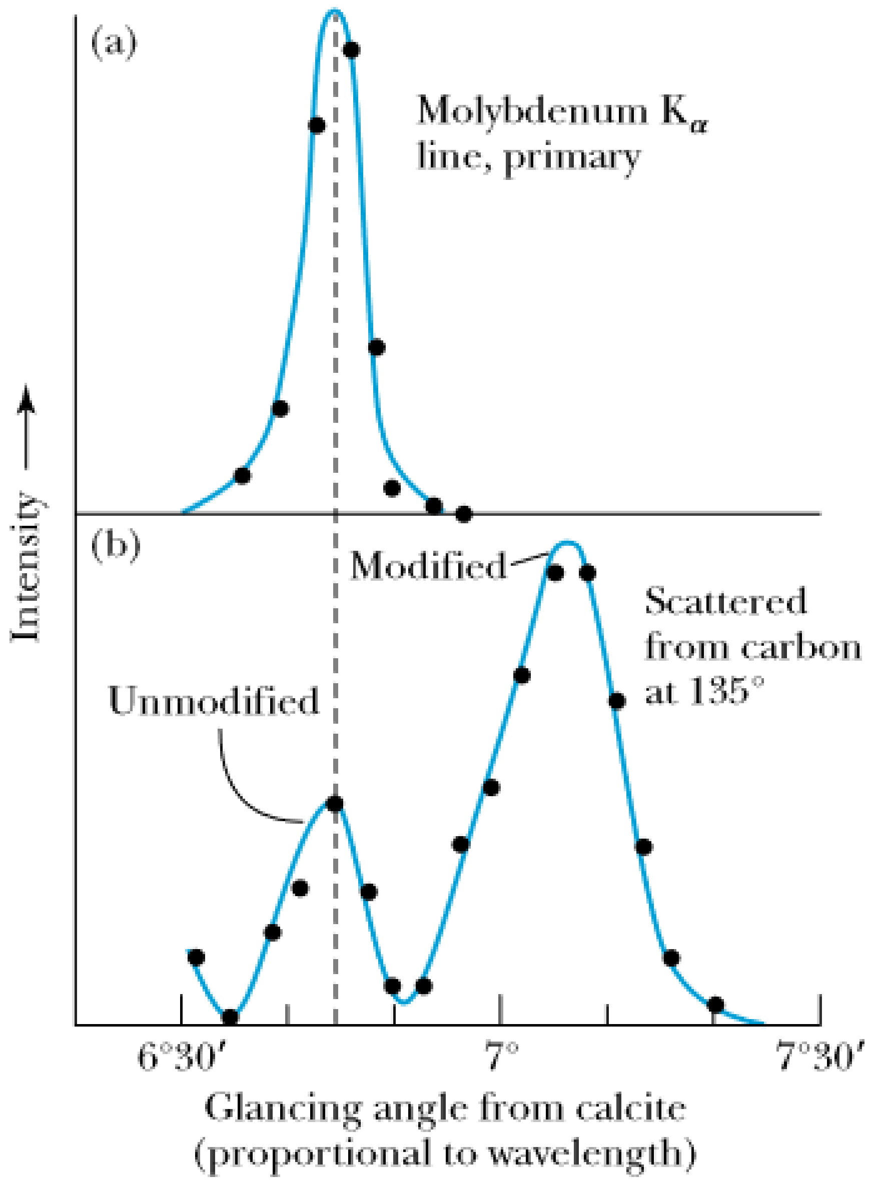
which is the result Compton found in 1923 for the **increase in wavelength of the scattered photon**.

Compton Effect

- Compton then proceeded to check the validity of his theoretical result by performing a careful experiment in which he **scattered X-rays of wavelength 0.071 nm from carbon at several angles**.

- He showed that the modified wavelength was in good agreement with his prediction.

- A part of his data is shown in Figure, where both the modified (λ’) and unmodified (λ) scattered waves are identified.



Compton’s original data showing (a) the primary x-ray beam from Mo unscattered and (b) the scattered spectrum from carbon at 135° showing both the modified and unmodified wave. Adapted from Arthur H. Compton, Physical Review 22, 409-413 (1923).

- The **kinetic energy and scattering angle of the recoiling electron** can also be calculated.
- Experiments in which the recoiling electrons were detected were soon carried out, thus completely **confirming Compton's theory**.
- **The process of elastic photon scattering from electrons is now called the Compton effect.**
- Note that the difference in wavelength, $\Delta\lambda = \lambda' - \lambda$, depends only on the constants *h, c*, and m_e in addition to the scattering angle θ.
- The quantity **$\lambda_C = h/m_e c$ 2.426 x 10^{-3} nm is called the Compton wavelength of the electron**.
- Only for wavelengths on the same order as λ_C (or shorter) will the fractional shift $\Delta\lambda/\lambda$ be large.
- For visible light, for example with $\lambda = 500$ nm, the maximum $\Delta\lambda/\lambda$ is on the order of 10^{-5} and $\Delta\lambda$ would be difficult to detect.
- The probability of the occurrence of the Compton effect for visible light is also quite small.
- However, **for X-rays of wavelength 0.071 nm used by Compton, the ratio of $\Delta\lambda/\lambda \approx 0.03$ and could easily be observed.**
- Thus, the **Compton effect is important only for X-rays or γ-ray photons** and is small for visible light.

Compton Effect

- The physical process of the Compton effect can be described as follows.
- The **photon elastically scatters from an essentially free electron** in the material.
- The photon’s energy is so much larger than the binding energy of the almost free electron that the **atomic binding energy can be neglected**.
- The newly created **scattered photon then has a modified, longer wavelength**.
- What happens if the photon scatters from one of the tightly bound inner electrons?
- Then the binding energy is not negligible, and the **electron might not be dislodged.**

- The scattering in this case is effectively from the **entire atom** (nucleus + electrons).
- Then the mass, is several thousand times larger than m_e, and Δλ is correspondingly smaller.
- Scattering from **tightly bound electrons** results in the unmodified photon scattering (λ ≈ λ'), which is also observed in Figure.
- **Thus, the quantum picture also explains the existence of the unmodified wavelength predicted by the classical theory (Thomson scattering) alluded to earlier.**
- **The success of the Compton theory convincingly demonstrated the correctness of both the quantum concept and the particle nature of the photon.**
- The use of the laws of the conservation of energy and momentum applied relativistically to pointlike scattering of the photon from the electron finally convinced the great majority of scientists of the validity of the new modern physics.
- Compton received the Nobel Prize in Physics for this discovery in 1927.

Pair Production and Annihilation

- A guiding principle of scientific investigation, if not a general rule of nature, is that if **some process is not absolutely forbidden** (by some law such as conservation of energy, momentum, or charge), then **we might expect that it will eventually occur**.
- In the photoelectric effect, bremsstrahlung, and the Compton effect, we have studied **exchanges of energy between photons and electrons**.
- Have we covered all possible mechanisms?
- For example, can the kinetic energy of a photon be converted into particle mass and vice versa?
- It would appear that if none of the conservation laws are violated, then such a process should be possible.
- First, let us consider the **conversion of photon energy into mass**.
- The electron, which has a mass ($m = 0.511\ \mathrm{MeV}/c^2$), is the lightest particle within an atom.

- If a **photon can create an electron**, **it must also create a positive charge to balance charge conservation.**
- In 1932, C. D. Anderson (Nobel Prize in Physics, 1936) observed a **positively charged electron (e^+)** in cosmic radiation.
- This particle, called a **positron**, had been predicted to exist several years earlier by P. A. M. Dirac (Nobel Prize in Physics, 1933).
- It has the **same mass as the electron but an opposite charge**.
- **Positrons are also observed when high-energy gamma rays (photons) pass through matter.**
- **Experiments show that a photon's energy can be converted entirely into an electron and a positron in a process called pair production.**

Pair Production and Annihilation

- The reaction is

$$\gamma \rightarrow e^{+} + e^{-}$$

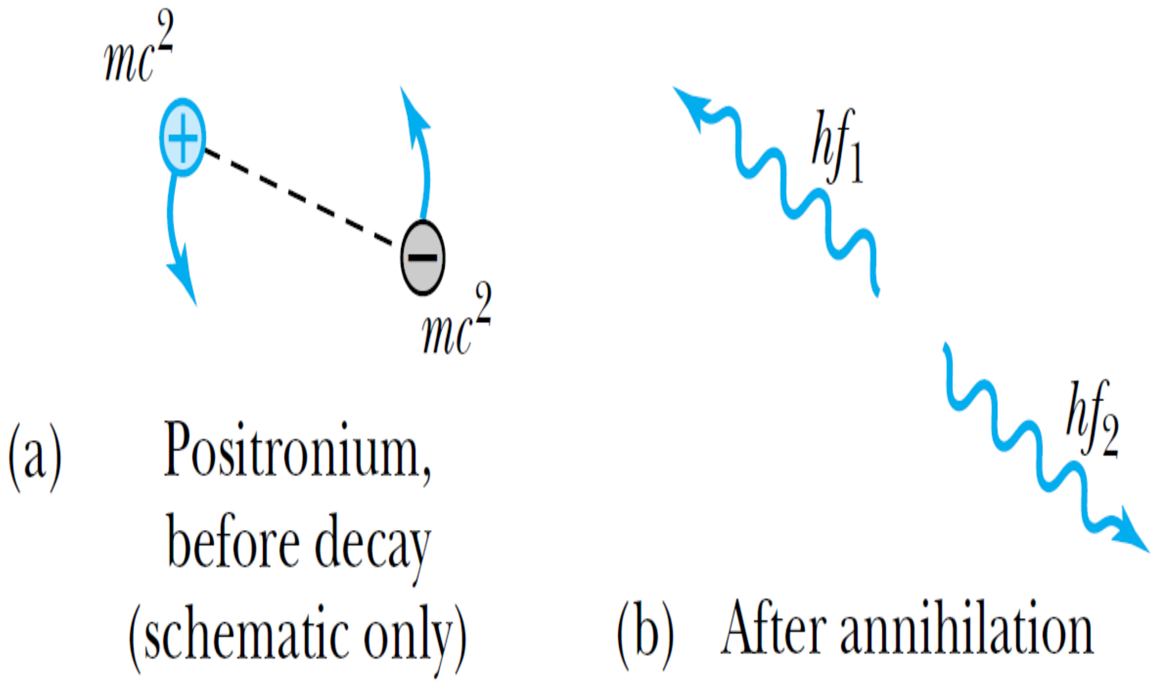
- However, **this process occurs only when the photon passes through matter**, because energy and momentum would not be conserved if the reaction took place in isolation.
- The missing momentum must be supplied by interaction with a nearby massive object such as a nucleus.
- Consider the conversion of a photon into an electron and a positron that takes place inside an atom where the electric field of a nucleus is large.
- The nucleus recoils and takes away a negligible amount of energy but a considerable amount of momentum. The conservation of energy will now be

$$hf = E_{+} + E_{-} + K.E.(nucleus)$$

Pair Production and Annihilation

- A diagram of the process is shown in Figure.
- The **photon energy must be at least equal to 2$\boldsymbol{m_e c^2}$** in order to create the masses of the electron and positron.

$$hf > 2m_e c^2 = 1{,}022\ MeV \qquad\qquad (for\ pair\ production)$$



Annihilation of positronium atom (consisting of an electron and positron), producing two photons.

- The **probability of pair production increases** dramatically both with **higher photon energy and with higher atomic number *Z* of the atom’s nucleus** because of the correspondingly higher electric field that mediates the process.
- Why positron is it not commonly found in nature?
- We also need to answer the question posed earlier: **can mass be converted to energy?**
- Positrons are **found in nature**.
- They are detected in cosmic radiation and as products of radioactivity from several radioactive nuclei.

Pair Production and Annihilation

- However, their existences are doomed because of their interaction with electrons.
- When **positrons and electrons** are in proximity for even a short time, they **annihilate each other, producing photons**.
- A **positron passing through matter** will quickly lose its kinetic energy through atomic collisions and will likely **annihilate with an electron**.
- After a **positron slows down**, it is drawn to an electron by their mutual electric attraction, and the electron and positron may then **form an atom like configuration called positronium**, in which they orbit around their common center of mass.
- Eventually the electron and positron annihilate each other (typically in 10^{-10} s), producing electromagnetic radiation (photons).
- The process $\boldsymbol{e^+ + e^- \rightarrow \gamma + \gamma}$ is called pair annihilation.

Pair Production and Annihilation

- Consider a positronium "atom" at rest in free space.
- It must **emit at least two photons to conserve energy and momentum**.
- If the positronium annihilation takes place near a nucleus, it **is possible that only one photon will be created**, because the missing momentum can be supplied by nucleus recoil as in pair production.
- Under certain conditions three photons may be produced. Because the emission of two photons is by far the most likely annihilation mode, let us consider this mode.
- The conservation laws for the process $(e^{+}e^{-})_{atom} \rightarrow \gamma + \gamma$ will be (we neglect the atomic binding energy of about 6.8 eV)

$$Energy \qquad 2m_e c^2 \approx hf_1 + hf_2$$

$$Momentum \qquad 0 = \frac{hf_1}{c} - \frac{hf_2}{c}$$

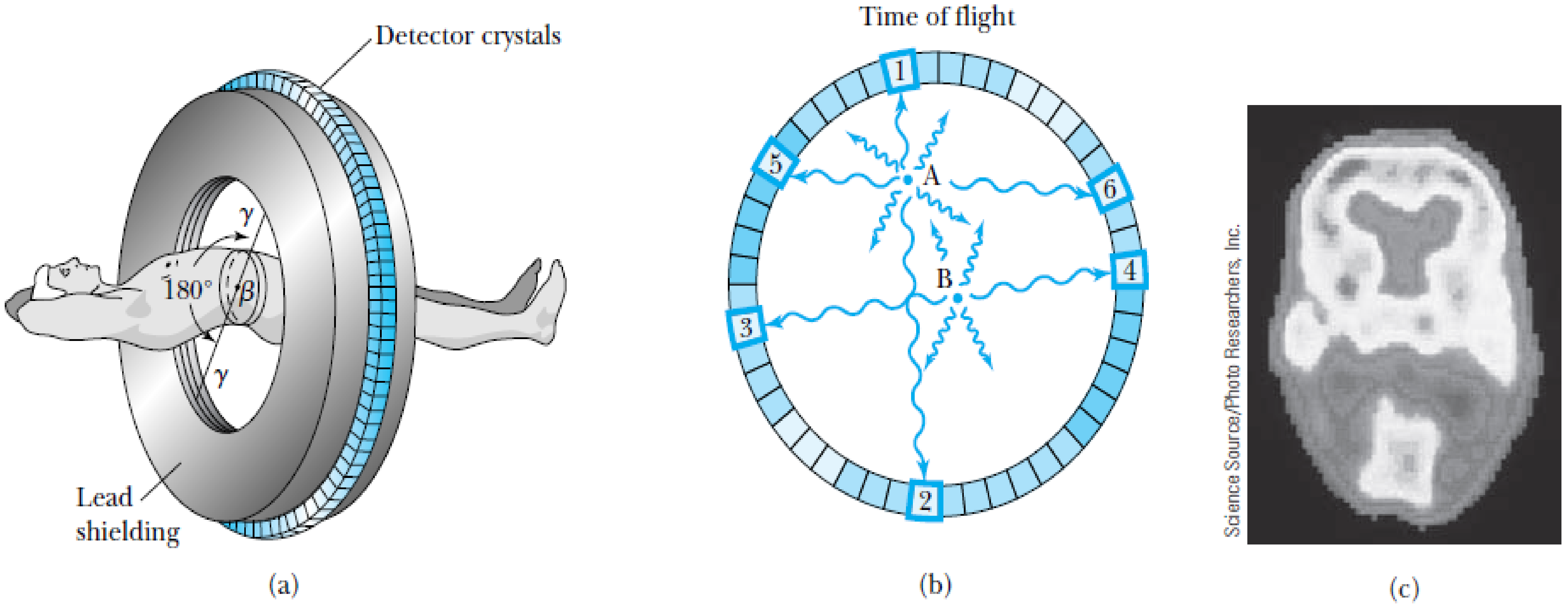
- The frequencies are identical, so we left $f_1 = f_2 = f$, thus becomes

$$2m_e c^2 = 2hf \qquad or \qquad hf = m_e c^2 = 0{,}511\ MeV$$

Pair Production and Annihilation

- In other words, the **two photons from positronium annihilation will move in opposite directions, each with energy 0.511 MeV.**
- This is exactly what is **observed experimentally**.
- The production of two photons in opposite directions with energies just over 0.5 MeV is so characteristic a signal of the presence of a positron that it has useful applications.
- Positron emission tomography (PET) scanning has become a standard diagnostic technique in medicine.
- A positron-emitting radioactive chemical (containing a nucleus such as ^{15}O, ^{11}C, ^{13}N, or ^{18}F) injected into the body causes two characteristic annihilation photons to be emitted from the points where the chemical has been concentrated by physiological processes.
- The location in the body where the photons originate is identified by measuring the directions of two gamma-ray photons of the correct energy that are detected in coincidence, as shown in Figure.
- Measurement of blood flow in the brain is an example of a diagnostic tool used in the evaluation of strokes, brain tumors, and other brain lesions.

Pair Production and Annihilation



Positron emission tomography (PET) is a useful medical diagnostic tool to study the path and location of a positron-emitting radiopharmaceutical in the human body.
(a) Appropriate radiopharmaceuticals are chosen to concentrate by physiological processes in the region to be examined.
(b) The positron travels only a few millimeters before annihilation, which produces two photons that can be detected to give the positron position.
(c) PET scan of a normal brain. *(a) and (b) are after G. L. Brownell et al.,* Science ***215****, 619 (1982)*

Pair Production and Annihilation

- **The positron is the antiparticle of the electron, having the opposite charge but the same mass**.
- In 1955 the antiproton was discovered by E. G. Segrè and O. Chamberlain (Nobel Prize, 1959), and today, many antiparticles are known.
- **We now believe that every particle has an antiparticle.**
- In some cases, as for photons or neutral pi-mesons, the particle and antiparticle are the same, but for most other particles, the particle and anti particle are distinct.
- For example, both the **neutron and proton have anitparticles called the antineutron and antiproton**.
- We know that matter and antimatter cannot exist together in our world, because their ultimate fate will be annihilation.
- If we believe in symmetry, might there not be another world, perhaps in a distant galaxy, that is made of antimatter?
- Because galaxies are so far apart in space, annihilation would be infrequent.
- However, if a large chunk of antimatter ever struck the Earth, it would tend to restore the picture of a symmetric universe.

EXAMPLES

EXAMPLE 3.11

Light of wavelength 400 nm is incident upon lithium (ϕ = 2.93 eV). Calculate (a) the photon energy and (b) the stopping potential V_0.

Strategy (a) Light is normally described by wavelengths in nm, so it is useful to have an equation to calculate the energy in terms of λ.

$$E = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^{8}\,\text{m/s})}{\lambda(1.602 \times 10^{-19}\,\text{J/eV})(10^{-9}\,\text{m/nm})}$$

$$\boxed{E = \frac{1.240 \times 10^{3}\,\text{eV}\cdot\text{nm}}{\lambda}} \tag{3.35}$$

(b) We use Equation (3.32) to determine the stopping potential once we know the frequency f and work function ϕ.

Solution (a) For a wavelength of λ = 400 nm we use Equation (3.35) to determine the photon's energy

$$E = \frac{1.240 \times 10^{3}\,\text{eV}\cdot\text{nm}}{400\,\text{nm}} = 3.10\,\text{eV}$$

(b) For the stopping potential, Equation (3.32) gives

$$eV_0 = hf - \phi = E - \phi = 3.10\,\text{eV} - 2.93\,\text{eV} = 0.17\,\text{eV}$$

$$V_0 = 0.17\,\text{V}$$

A retarding potential of 0.17 V will stop all photoelectrons.

EXAMPLE 3.12

(a) What frequency of light is needed to produce electrons of kinetic energy 3.00 eV from illumination of lithium?
(b) Find the wavelength of this light and discuss where it is in the electromagnetic spectrum.

Strategy We have enough information to determine the photon energy needed from Equation (3.30), and we can determine the frequency from $E = hf$.

Solution From Equation (3.30), we have

$$\begin{aligned} hf &= \phi + \tfrac{1}{2} m v_{\max}^2 \\ &= 2.93\ \text{eV} + 3.00\ \text{eV} = 5.93\ \text{eV} \end{aligned}$$

The photon frequency is now found to be

$$\begin{aligned} f = \frac{E}{h} &= \frac{(5.93\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})}{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})} \\ &= 1.43 \times 10^{15}\ \text{s}^{-1} = 1.43 \times 10^{15}\ \text{Hz} \end{aligned}$$

(b) The wavelength of the light can be found from $c = \lambda f$.

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\ \text{m/s}}{1.43 \times 10^{15}\ \text{Hz}} = 2.10 \times 10^{-7}\ \text{m} = 210\ \text{nm}$$

This is ultraviolet light, because the wavelength 210 nm is below the range of visible wavelengths 400 to 700 nm.

EXAMPLE 3.15

If we have a tungsten anode (work function $\phi = 4.63$ eV) and electron acceleration voltage of 35 kV, why do we ignore in Equation (3.36) the initial kinetic energy of the electrons from the filament and the work functions of the filaments and anodes? What is the minimum wavelength of the x rays?

Strategy We can ignore the initial electron kinetic energies and the work functions, because they are on the order of a few electron volts (eV), whereas the kinetic energy of the electrons due to the accelerating voltage is 35,000 eV. The error in neglecting everything but eV_0 is small. We will use Equation (3.37) to determine the minimum wavelength.

Solution We use the Duane-Hunt rule of Equation (3.37) to determine

$$\lambda_{\min} = \frac{1.240 \times 10^{-6}\ \mathrm{V \cdot m}}{35.0 \times 10^{3}\ \mathrm{V}} = 3.54 \times 10^{-11}\ \mathrm{m}$$

which is in good agreement with the data of Figure 3.19.

EXAMPLE 3.16

An x ray of wavelength 0.050 nm scatters from a gold target. (a) Can the x ray be Compton-scattered from an electron bound by as much as 62 keV? (b) What is the largest wavelength of scattered photon that can be observed? (c) What is the kinetic energy of the most energetic recoil electron and at what angle does it occur?

Strategy We first determine the x-ray energy to see if it has enough energy to dislodge the electron. We use Equation (3.42) with both the atomic and electron mass to determine the scattered photon wavelength. We then use the conservation of energy to determine the recoil electron kinetic energy.

Solution From Equation (3.35) the x-ray energy is

$$E_{\text{x ray}} = \frac{1.240 \times 10^3\ \text{eV}\cdot\text{nm}}{0.050\ \text{nm}} = 24{,}800\ \text{eV} = 24.8\ \text{keV}$$

Therefore, the x ray does not have enough energy to dislodge the inner electron, which is bound by 62 keV. In this case we have to use the atomic mass in Equation (3.42), which results in little change in the wavelength (Thomson scattering).

Scattering may still occur from outer electrons, so we examine Equation (3.42) with the electron mass. The longest wavelength $\lambda' = \lambda + \Delta\lambda$ occurs when $\Delta\lambda$ is a maximum or when $\theta = 180°$.

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos 180°) = \lambda + \frac{2h}{m_e c}$$

$$= 0.050 \text{ nm} + 2(0.00243 \text{ nm}) = 0.055 \text{ nm}$$

The energy of the scattered photon is then a minimum and has the value

$$E'_{\text{x ray}} = \frac{1.240 \times 10^3 \text{ eV} \cdot \text{nm}}{0.055 \text{ nm}} = 2.25 \times 10^4 \text{ eV} = 22.5 \text{ keV}$$

The difference in energy of the initial and final photon must equal the kinetic energy of the electron (neglecting binding energies). The recoil electron must scatter in the forward direction at $\phi = 0°$ when the final photon is in the backward direction ($\theta = 180°$) to conserve momentum. The kinetic energy of the electron is then a maximum.

$$E_{\text{x ray}} = E'_{\text{x ray}} + \text{K.E. (electron)}$$

$$\text{K.E. (electron)} = E_{\text{x ray}} - E'_{\text{x ray}}$$

$$= 24.8 \text{ keV} - 22.5 \text{ keV} = 2.3 \text{ keV}$$

Because $\Delta\lambda$ does not depend on λ or λ', we can determine the wavelength (and energy) of the incident photon by merely observing the kinetic energy of the electron at forward angles (see Problem 60).

EXAMPLE 3.17

Show that a photon cannot produce an electron-positron pair in free space as shown in Figure 3.22a.

Strategy We need to look carefully at the conservation of momentum and energy to see whether pair production can occur in free space.

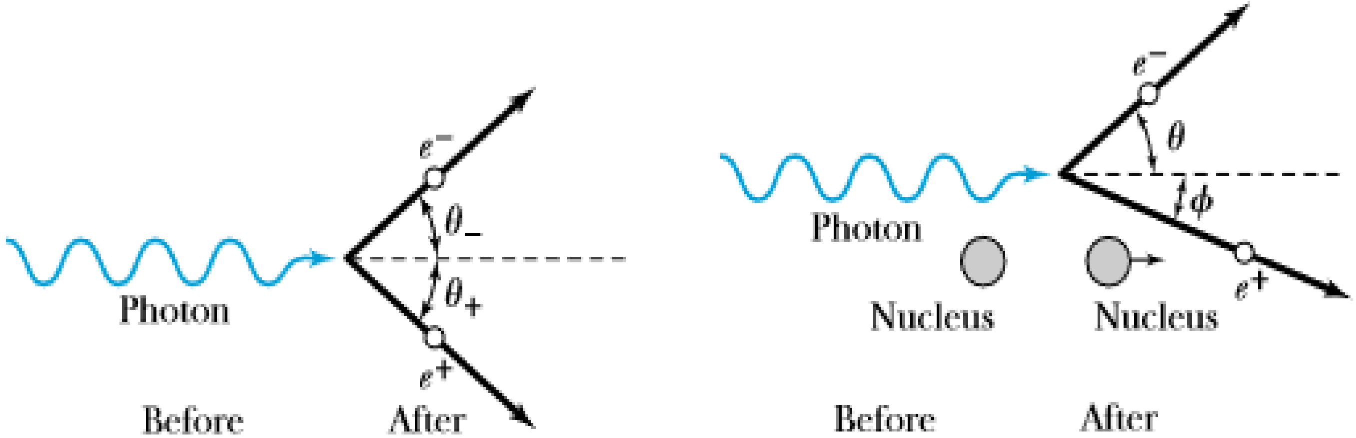
Solution Let the total energy and momentum of the electron and the positron be E_-, p_- and E_+, p_+, respectively. The conservation laws are then

$$\text{Energy} \qquad hf = E_+ + E_- \tag{3.44a}$$

$$\text{Momentum, } p_x \qquad \frac{hf}{c} = p_- \cos\theta_- + p_+ \cos\theta_+ \tag{3.44b}$$

$$\text{Momentum, } p_y \qquad 0 = p_- \sin\theta_- - p_+ \sin\theta_+ \tag{3.44c}$$

Figure 3.22 (a) A photon cannot decay into an electron-positron pair in free space, but (b) if a nucleus is nearby, the nucleus can absorb sufficient linear momentum to allow the process to proceed.



(a) Free space (**cannot occur**) (b) Beside nucleus

Equation (3.44b) can be written as

$$hf = p_-c\cos\theta_- + p_+c\cos\theta_+ \tag{3.45}$$

If we insert $E_\pm^2 = p_\pm^2c^2 + m^2c^4$ into Equation (3.44a), we have

$$hf = \sqrt{p_+^2c^2 + m^2c^4} + \sqrt{p_-^2c^2 + m^2c^4} \tag{3.46}$$

The maximum value of hf is, from Equation (3.45),

$$hf_{\text{max}} = p_-c + p_+c$$

However, from Equation (3.46), we also have

$$hf > p_-c + p_+c$$

Equations (3.45) and (3.46) are inconsistent and cannot simultaneously be valid. Equations (3.44), therefore, do not describe a possible reaction. The reaction displayed in Figure 3.22a is not possible, because energy and momentum are not simultaneously conserved.