Structure of the Atom

- ✓Scientists of the late nineteenth century did not have technology to see things **as small as atoms**, they believed that **atoms were composite** structures having an internal structure.

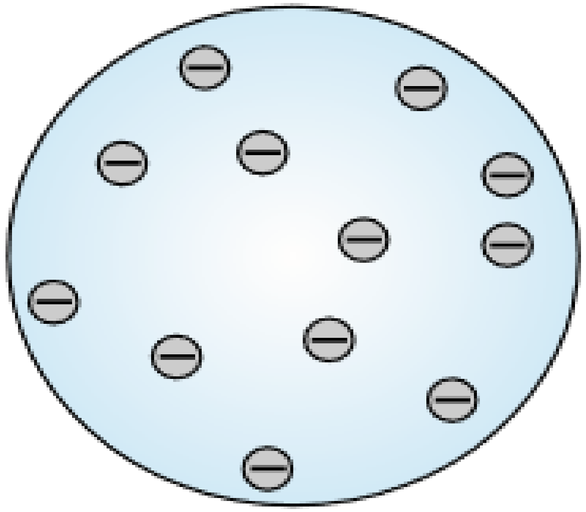
- ✓It was found **experimentally** that **atoms and electromagnetic** phenomena were intimately related.

- ✓In order to make further progress in deciphering atomic structure, a new approach was needed.

- ✓The new direction was supplied by Ernest **Rutherford**, who was already famous for his Nobel Prize– winning work on **radioactivity**.

The Atomic Models of Thomson and Rutherford

- ✓**Thomson** proposed a model wherein the **positive** charges were spread uniformly throughout a sphere the size of the atom, with **electrons** embedded in the uniform background.
- ✓ His model, which was likened to **raisins in plum pudding**.
- ✓The **arrangement of charges** had to be in stable **equilibrium**.
- ✓In **Thomson's** view, when the atom was heated, the electrons could vibrate about their equilibrium positions, thus producing electromagnetic radiation.
- ✓The emission frequencies of this radiation would fall in the range of **visible light** if the sphere of positive charges were of diameter **~10^{-10} m**, which was known to be the **approximate size of an atom**.
- ✓**Thomson** was unable to calculate the light spectrum of hydrogen using his model.



Schematic of J. J. Thomson's model of the atom (**later proved to be incorrect**).

The electrons are embedded in a homogeneous positively charged mass much like raisins in plum pudding. The electric force on the electrons is zero, so the electrons do not move around rapidly. The oscillations of the electrons give rise to electromagnetic radiation.

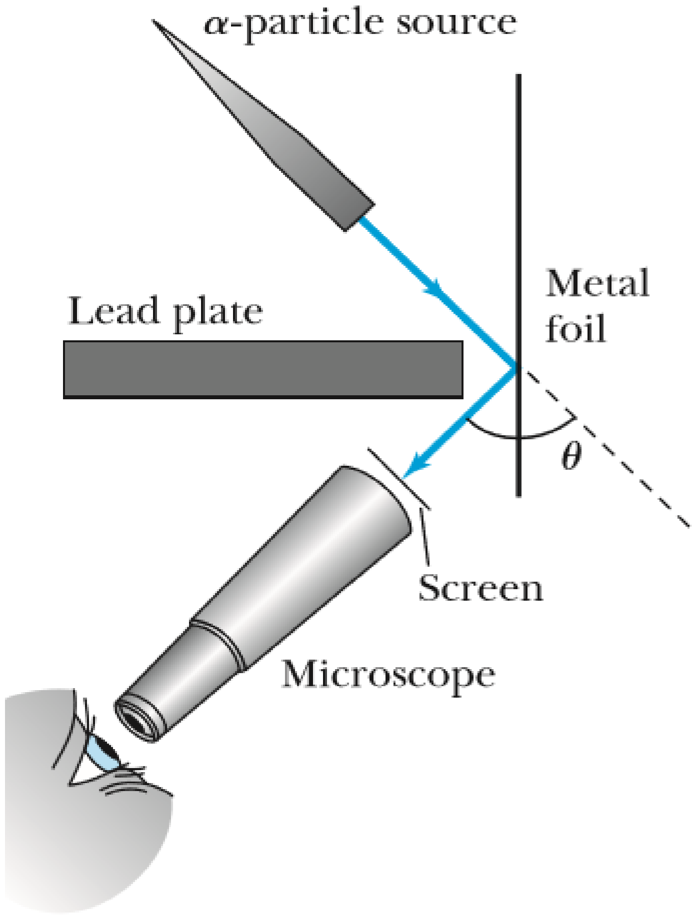
✓The **small size of the atoms** made it **impossible to see** directly their internal structure.

✓**Rutherford's assistant Geiger with a student Marsden,** projected very small particles onto thin material, some of which collided with atoms and eventually exited **at various angles**.

✓ Many **α particles were scattered** from thin gold-leaf targets at backward angles greater than 90°.

✓Rutherford could see that Thomson's model **agreed neither with spectroscopy nor with Geiger's latest experiment** with α particles.

- Schematic diagram of apparatus used by Geiger and Marsden to observe scattering of α particles past 90°.
- "A small fraction of the α particles falling upon a metal foil have their directions changed to such an extent that they emerge again at the side of incidence."
- **The scattered α particle struck a scintillating screen where the brief flash was observed through the microscope.**



- ✓The experiments of Geiger and Marsden were instrumental in the development of Rutherford’s model.
- ✓ A **simple thought experiment** with a .22-caliber rifle that fires a bullet into a thin black box is a model for understanding the problem.
- ✓If the box contains a homogeneous material such as wood or water (as in Thomson’s plum-pudding model), the bullet will pass through the box with **little or no deviation** in its path.
- ✓However, if the box contains a few massive steel ball bearings, then occasionally **a bullet will be deflected backward**, similar to what Geiger and Marsden observed with a scattering.

Multiple scattering from electron

- ✓What would happen if an $\propto$ **particle** were scattered by many electrons in the target?
- ✓**Multiple scattering is possible**, and a calculation for random multiple scattering from **N electrons** results in **an average scattering angle**

$$< \theta >_{total} \approx \sqrt{N}\theta.$$

- ✓The $\propto$ **particle** is as likely to scatter on one side of its direction as the other side for each collision.
- ✓We can estimate **the number of atoms across the thin gold layer** of $6x10^{-7}$ m used by Geiger and Marsden.

Mass of Gold metal

$$\frac{\text{Number of molecules}}{\text{cm}^3} = \left(6.02 \times 10^{23}\ \frac{\text{molecules}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{197\ \text{g}}\right)\left(19.3\ \frac{\text{g}}{\text{cm}^3}\right)$$

$$= 5.9 \times 10^{22}\ \frac{\text{molecules}}{\text{cm}^3} = 5.9 \times 10^{28}\ \frac{\text{atoms}}{\text{m}^3}$$

cubic centimeter (cc, cm3) of gold

- ✓ If there are 5.9 x 10^{28} atoms/m^3, then each atom occupies (5.9 x 10^{28})$^{-1}$ m^3 of space.
- ✓ Assuming the **atoms are equidistant**, the distance **d between centers** is
  d = (5.9 x 10^{28})$^{-1/3}$ m = 2.6 x 10^{-10} m.
- ✓ In the foil, then, there are

$$N = \frac{6 \times 10^{-7}\ \text{m}}{2.6 \times 10^{-10}\ \text{m}} = 2300\ \text{atoms}$$

along the α particle’s path.

✓If we assume the **α particle interacts with one electron** from each atom, then

$$\langle\theta\rangle_{\text{total}} = \sqrt{2300}\,(0.016°) = 0.8°$$

This result can be derived by using $\Delta\vec{p}_\alpha = M_\alpha\vec{v}_\alpha - M_\alpha\vec{v}'_\alpha = m_e\vec{v}'_e$

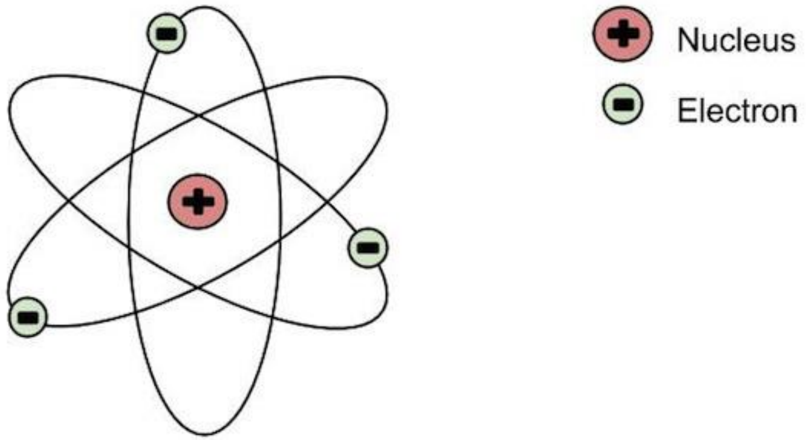
In an elastic collision with such unequal masses, $\mathbf{v_e' \approx 2\,v_\alpha}$.

And $\theta_{\max} = \dfrac{\Delta p_\alpha}{p_\alpha} = \dfrac{2m_e v_\alpha}{M_\alpha v_\alpha} = \dfrac{2m_e}{M_\alpha} = 2.7 \times 10^{-4}\,\text{rad} = 0.016°$

✓Even if the α particle scattered from all **79 electrons** in each atom of gold, $<\theta>_{total}$ = 6.8°.

- ✓**Rutherford** reported in 1911 that the experimental results were **not consistent** with α-particle scattering from the atomic structure proposed by Thomson and that "it seems reasonable to suppose that the deflection through a large angle is due to a **single atomic encounter**."
- ✓**Rutherford proposed that an atom consisted mostly of empty space with a central charge, either positive or negative.**
- ✓**Rutherford** wrote in 1911, "Considering the evidence as a whole, it seems simplest to suppose that the atom **contains a central charge** distributed through a very small volume, and that the large single deflections are due to the **central charge as a whole**, and not to its constituents."
- ✓**Rutherford** worked out the scattering expected for the ∝ particles as a function of angle, thickness of material, velocity, and charge.
- ✓**Geiger and Marsden** immediately began an experimental investigation of Rutherford's ideas and reported in 1913, "**we have completely verified the theory given by Prof. Rutherford**."
- ✓In that same year, **Rutherford was the first to use the word nucleus** for the central charged core and definitely decided that the core (containing most of the mass) was positively charged, surrounded by the negative electrons.

- ✓The popular conception of an atom today, often depicted as in Figure, is due to Rutherford.



Rutherford Model of the Atom

- ✓An extremely **small positively charged core** provides a Coulomb attraction for the negatively charged electrons flying at high speeds around the nucleus; this is the “solar system” or “planetary” model.
- ✓**We now know that the nucleus is composed of positively charged protons and neutral neutrons, each having approximately the same mass, and the electrons do not execute prescribed orbital paths.**

Rutherford Scattering

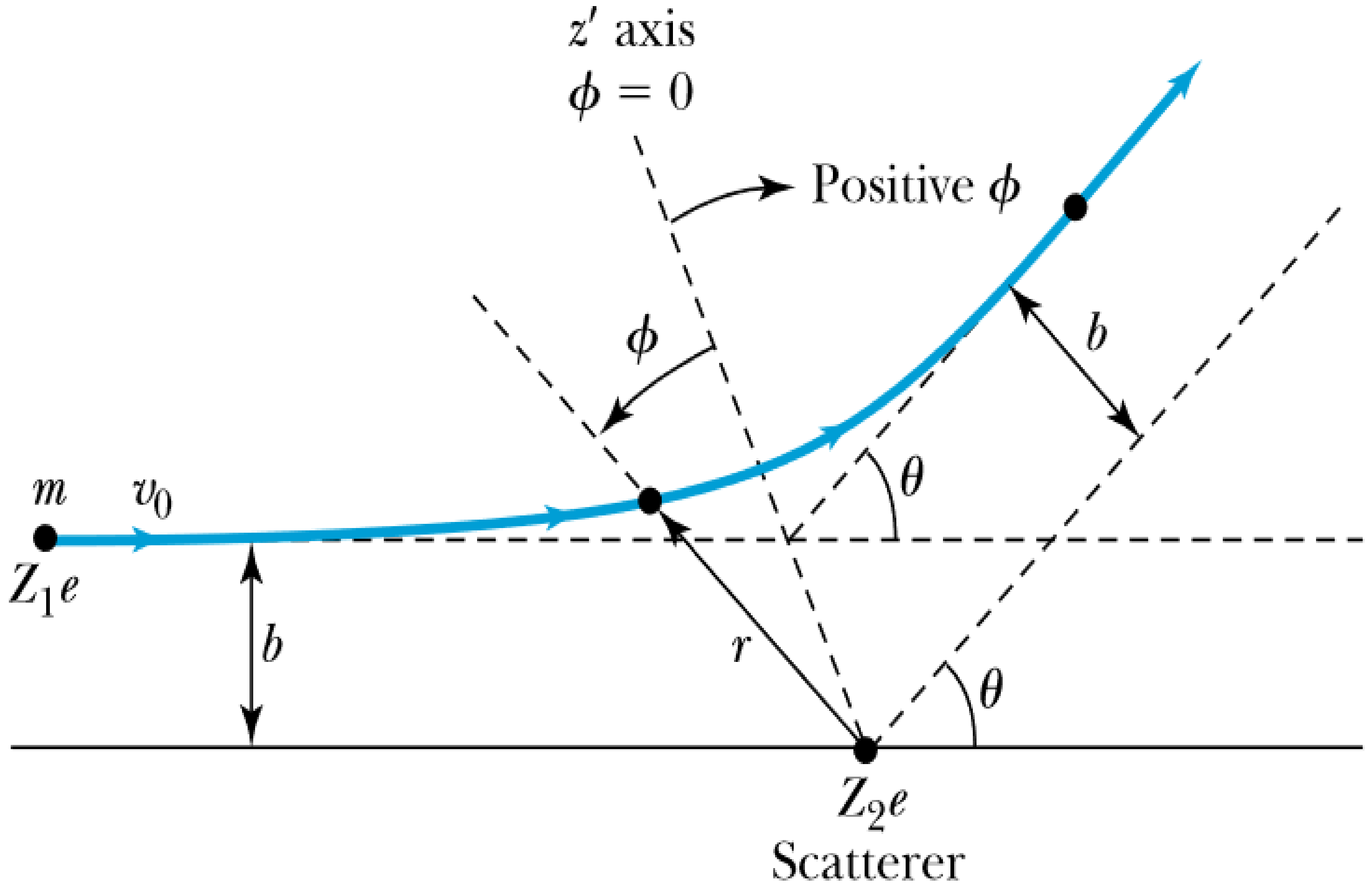
- ✓Rutherford's "**discovery of the nucleus**" laid the foundation for many of today's atomic and nuclear scattering experiments.

- ✓**Scattering experiments help us study matter** on an atomic scale, which is too small to be observed directly.

- ✓The material to be studied is bombarded with rapidly moving particles (such as the 5- to 8-MeV α particles used by Geiger and Marsden) in a well-defined and collimated beam.

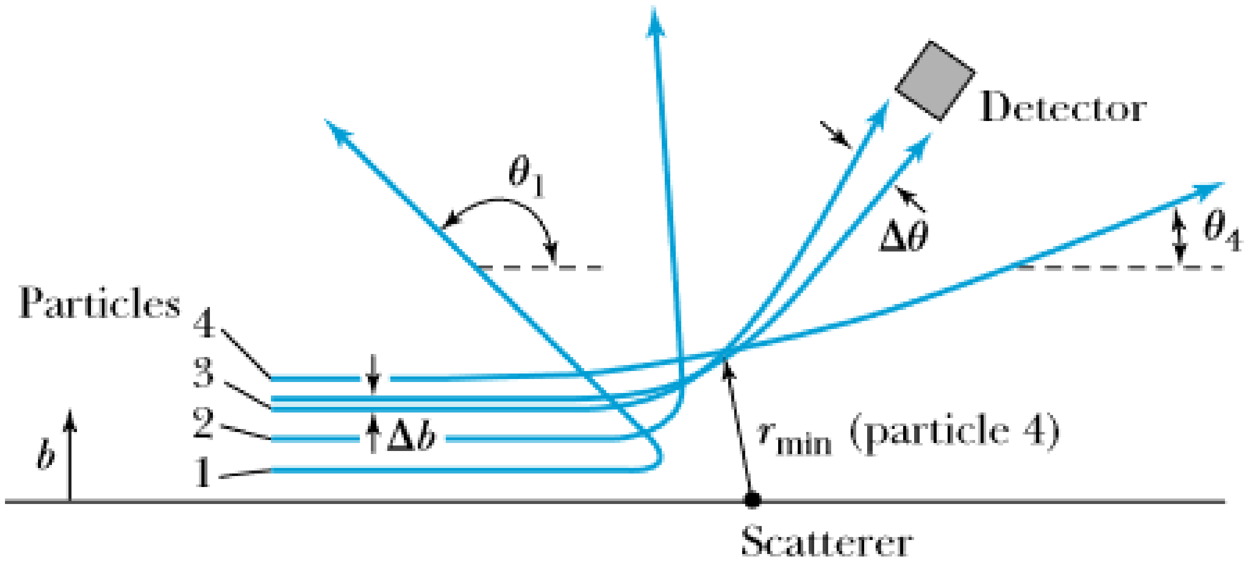
- ✓Although the present discussion is limited to **charged-particle beams,** the general procedure also applies to **neutral particles** such as **neutrons**; only the interaction between the beam particles and the target material is different.
- ✓The scattering of charged particles by matter is **called Coulomb or Rutherford scattering** when it takes place at **low energies**, where only the Coulomb force is important.
- ✓At **higher beam energies** other forces (for example, nuclear interactions) may become important.

- ✓ A charged particle of mass m, charge Z_1e, and speed v_0 is incident on the target material or scatterer of charge Z_2e.
- ✓ The distance **b is called the classical impact parameter**; it is the closest distance of approach between the beam particle and scatterer if the projectile had continued in a straight line.
- ✓ The angle **θ** between the incident beam direction and the direction of the deflected particle is called the **scattering angle**.



✓A typical scattering experiment is diagrammed in Figure.

- Normally detectors are positioned at one or more scattering angles to count the particles scattered into the **small cones of solid angle** subtended by the detectors.



- Depending on the functional form of the interaction between the particle and the scatterer, there will be a particular relationship between the **impact parameter b and the scattering angle θ**.
- In the case of **Coulomb scattering** between a positively charged α particle and a positively charged nucleus, the trajectories resemble those in this Figure.

- When the **impact parameter is small**, the distance of **closest approach r_{min} is small**, and the Coulomb force is large.
- Therefore, the **scattering angle is large**, and the particle is **repelled** backward.
- Conversely, for **large impact parameters** the particles **never get close** together, so the **Coulomb force is small** and the **scattering angle is also small.**
- An important relationship for any interaction is that between **b and θ**.
- We wish to find this dependence for the Coulomb force.
- We will make the same assumptions as Rutherford.

Scattering assumptions

1. The **scatterer** is so massive that it does not significantly recoil; therefore the initial and final kinetic energies of the a particle are practically equal.

2. The **target is so thin** that only a single scattering occurs.

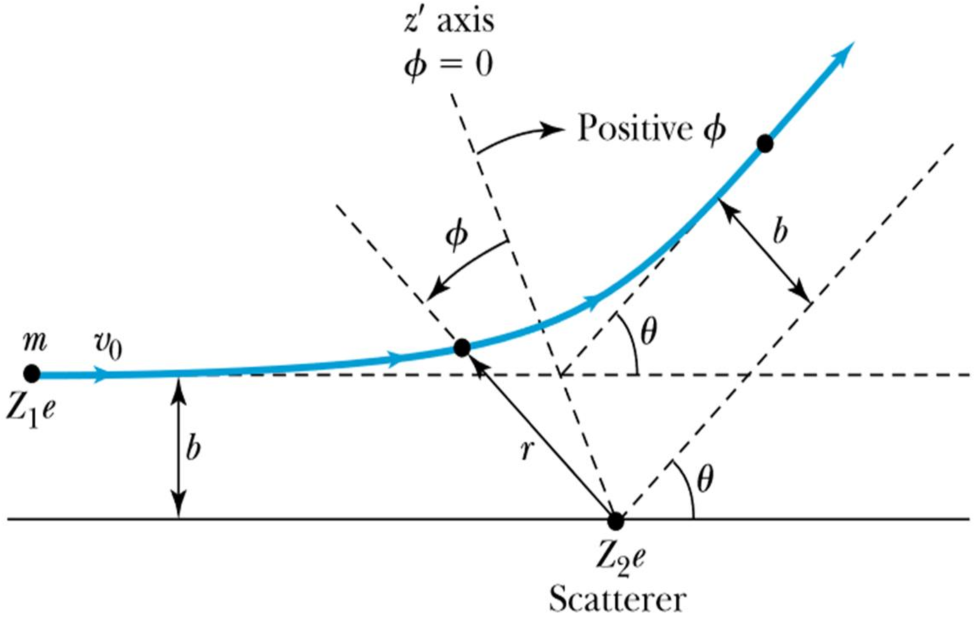
3. The **bombarding particle** and **target scatterer** are so **small** that they may be treated as **point masses and charges**.

4. Only the **Coulomb force** is effective.

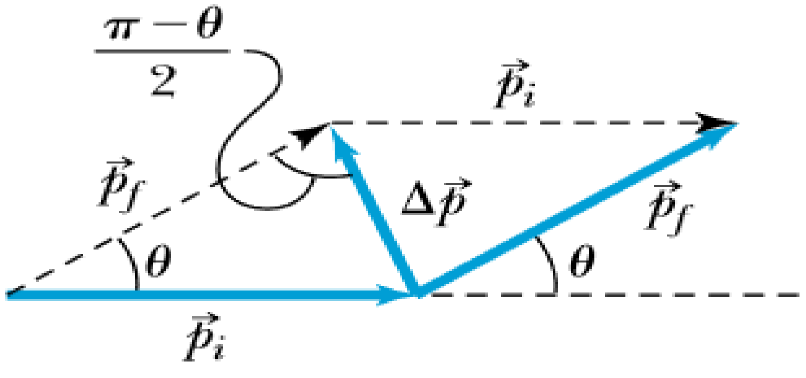
- Assumption 1 means that **K ≡ K.E.$_{\text{initial}}$ ≈ K.E.$_{\text{final}}$** for the α particle.
- For **central forces such as the Coulomb force**, the angular momentum, mv_0b, where v_0 is the initial velocity of the particle, is also **conserved**.
- This means that the **trajectory** of the scattered particle lies in a plane.
- We define the **instantaneous position of the particle by the angle ϕ** and the distance r from the force center, where ϕ = 0 (which defines the z′ axis) when the distance r is a minimum, as shown in Figure.
- The **change in momentum** is equal to the **impulse**.

$$\Delta\vec{p} = \int \vec{F}_{\Delta p}\, dt$$

where $\overrightarrow{F}_{\Delta p}$ is the force along the direction of $\Delta\vec{p}$.



- The **massive scatterer absorbs** this (small) **momentum change** without gaining any appreciable kinetic energy (no recoil).
- We use the diagram of to show

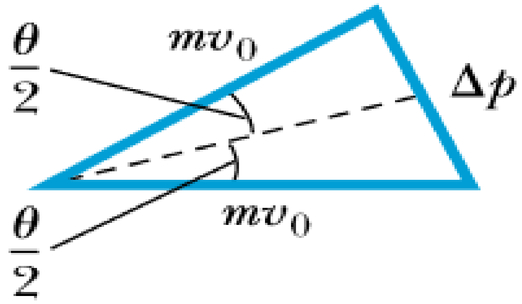


(a)

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

where the subscripts **i and f indicate the initial and final** values of the projectile's momentum, respectively.

- Because $p_f \approx p_i = mv_0$, the triangle between $\vec{p}_f$, $\vec{p}_i$, and $\Delta\vec{p}$ is isosceles.
- We redraw the triangle in Figure b, indicating the bisector of angle θ.
- The **magnitude Δp** of the vector $\Delta\vec{p}$ is now



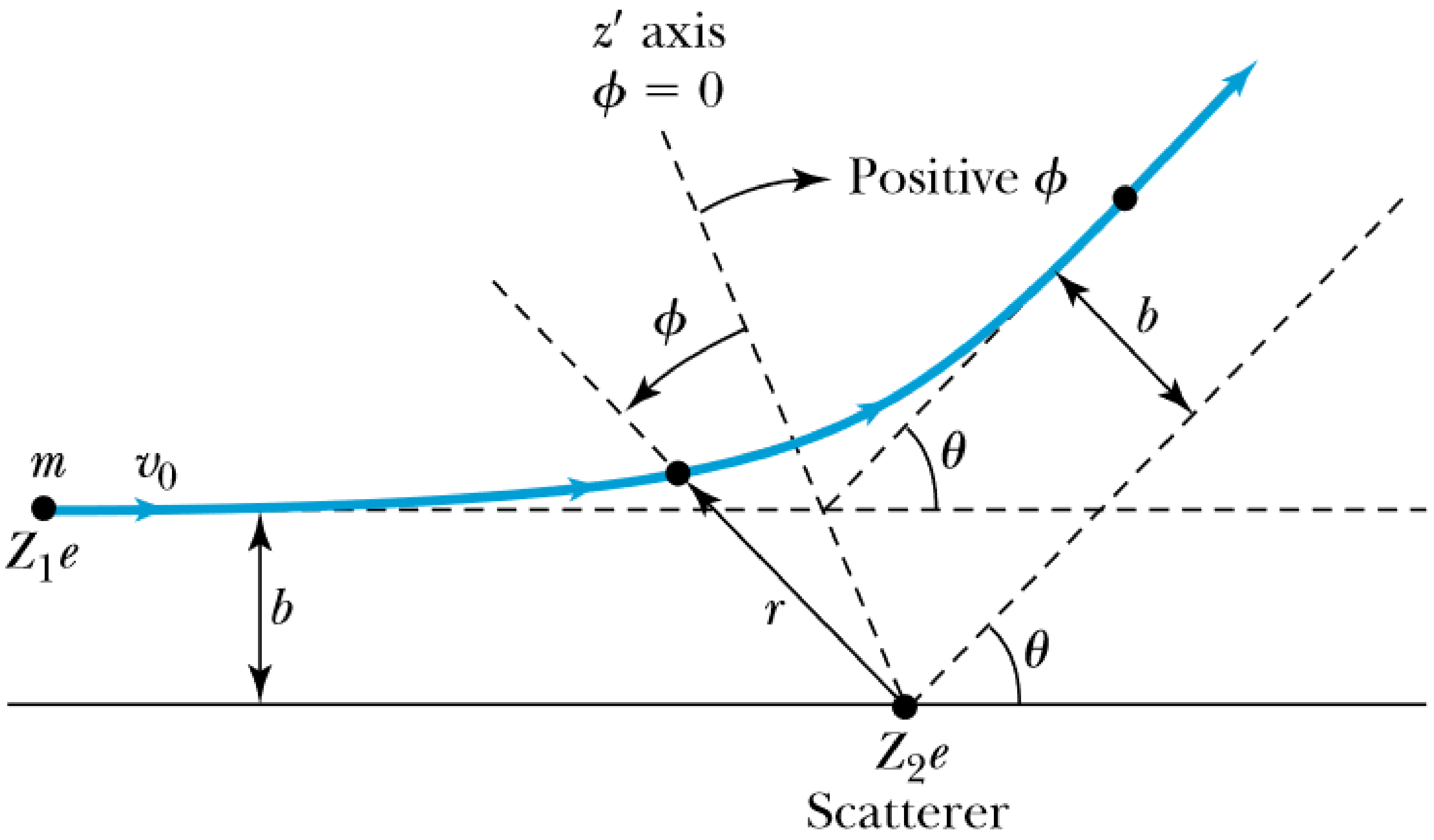
(b)

$$\Delta p = 2mv_0 \sin\frac{\theta}{2}$$

- The direction of $\Delta\vec{p}$ is the z' axis (where ϕ = 0), so we need the component of $\vec{F}$ along z' axis in the equation.
- The **Coulomb force $\vec{F}$** is along the instantaneous direction of the position vector $\vec{r}$ (unit vector $\hat{e}_r$, where ˆ indicates a unit vector).

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{r^2}\hat{e}_r$$

$$F_{\Delta p} = F\cos\phi$$



- $F_{\Delta p}$ is the component of the force $\vec{F}$ along the direction of $\Delta\vec{p}$ that we need.

$$\Delta p = 2mv_0 \sin\frac{\theta}{2} = \int F\cos\phi\, dt = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\int \frac{\cos\phi}{r^2}\, dt$$

The instantaneous angular momentum must be conserved, so

$$mr^2 \frac{d\phi}{dt} = mv_0 b$$

and

$$r^2 = \frac{v_0 b}{d\phi / dt}$$

Therefore,

$$2mv_0 \sin\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\int \frac{\cos\phi}{v_0 b}\frac{d\phi}{dt}\, dt$$

$$= \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 v_0 b}\int_{\phi_i}^{\phi_f} \cos\phi\, d\phi$$

We let the initial angle ϕ_i be on the negative side and the final angle ϕ_f be on the positive side of the z' axis ($\phi = 0$, see Figure). Then we have $\phi_i = -\phi_f$, and $-\phi_i + \phi_f + \theta = \pi$, so $\phi_i = -(\pi - \theta)/2$ and $\phi_f = +(\pi - \theta)/2$.

$$\frac{8\pi\epsilon_0 m v_0^2 b}{Z_1 Z_2 e^2} \sin\frac{\theta}{2} = \int_{-(\pi-\theta)/2}^{+(\pi-\theta)/2} \cos\phi \, d\phi = 2\cos\frac{\theta}{2}$$

We now solve this equation for the impact parameter b.

$$b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 m v_0^2} \cot\frac{\theta}{2}$$

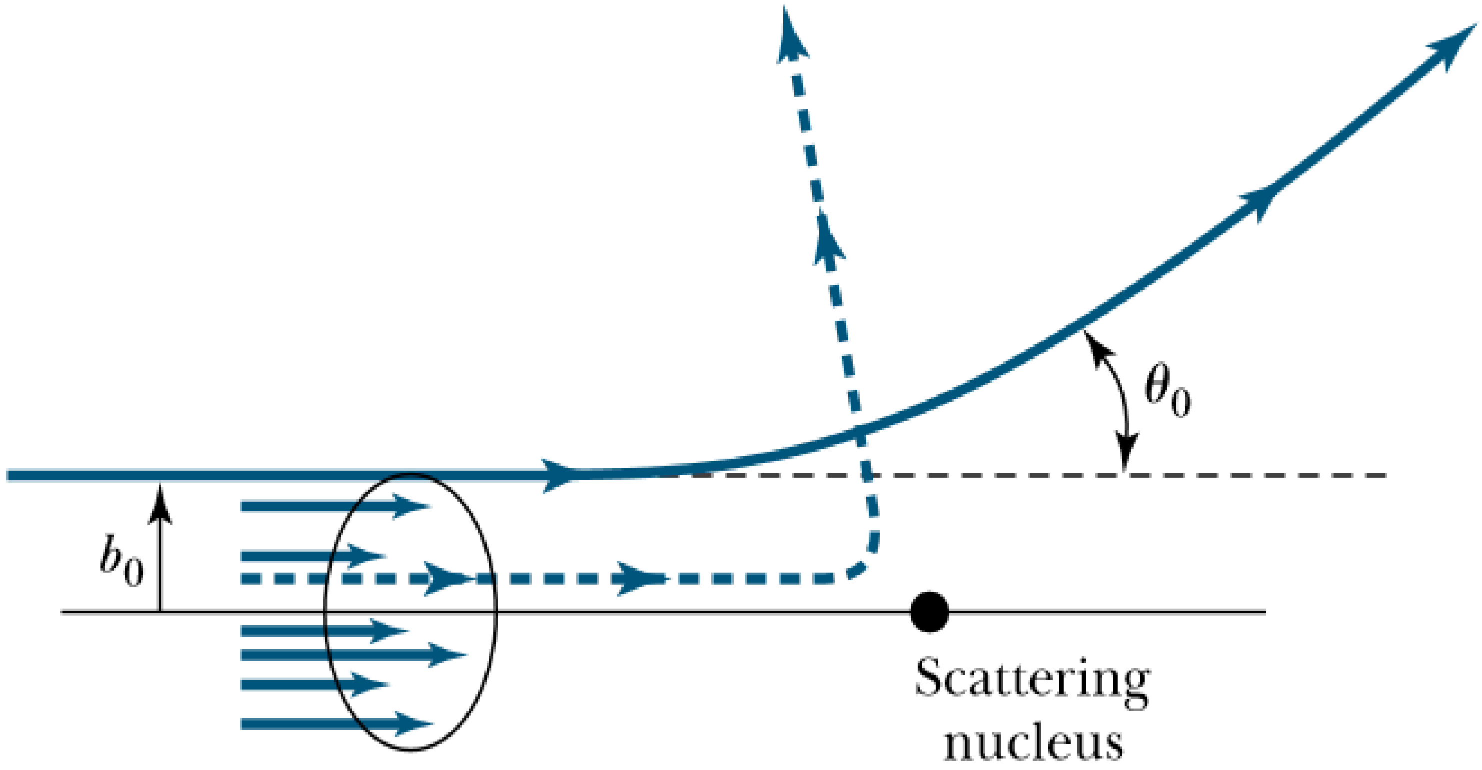
This equation becomes

Relation between b and θ

$$b = \frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K} \cot\frac{\theta}{2}$$

where $K = mv_0^2/2$ is the **kinetic energy** of the bombarding particle.

- This is the **fundamental relationship between the impact parameter b and scattering angle θ** that we have been seeking for the Coulomb force.
- We are **not able to select** individual impact parameters **b** in a given experiment.
- When we put a detector at a particular angle θ, we cover a finite **Δθ**, which corresponds to a range of impact parameters **Δb**.
- The bombarding particles are incident at varied impact parameters all around the scatterer.



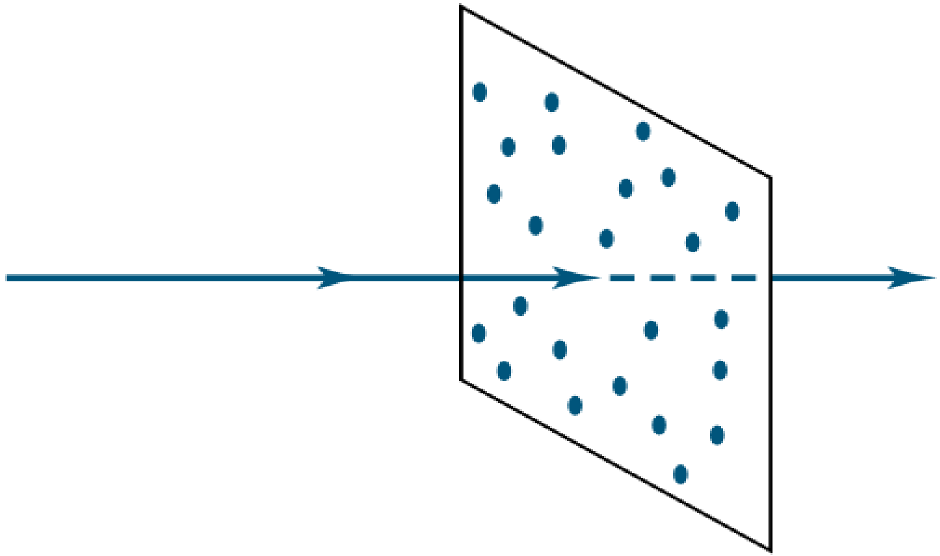
- All the particles with impact parameters **less than b_0** will be scattered at angles **greater than θ_0**.
- Any particle with an impact parameter inside the area of the circle of area πb_0^2 (radius b_0) will be similarly scattered.
- For the case of Coulomb scattering, we denote the **cross section** by **the symbol σ**, where

$$\sigma = \pi b^2$$

is the cross section for scattering through an angle θ or more.

- The cross section **σ** is related to the **probability** for a particle being scattered by a nucleus.

- If we have a target foil of thickness t with n atoms/volume, the number of target nuclei per unit area is **nt**.
- Because we assumed a thin target of area A and all nuclei are exposed as shown in Figure, the number of target nuclei is simply **ntA**.



- The value of n is the density ρ (g/cm^3) times Avogadro's number N_A (molecules/ mol) times the number of atoms/molecule N_M divided by the gram-molecular weight M_g (g/mol).

$$n = \frac{\rho\left(\dfrac{\text{g}}{\text{cm}^3}\right) N_\text{A}\left(\dfrac{\text{molecules}}{\text{mol}}\right) N_M\left(\dfrac{\text{atoms}}{\text{molecule}}\right)}{M_g\left(\dfrac{\text{g}}{\text{mol}}\right)} = \frac{\rho N_\text{A} N_M}{M_g}\,\frac{\text{atoms}}{\text{cm}^3}$$

The number of scattering nuclei per unit area is nt.

$$nt = \frac{\rho N_\text{A} N_M t}{M_g}\,\frac{\text{atoms}}{\text{cm}^2}$$

If we have a foil of area A, the number of target nuclei N_s is

$$N_s = ntA = \frac{\rho N_\text{A} N_M tA}{M_g}\ \text{atoms}$$

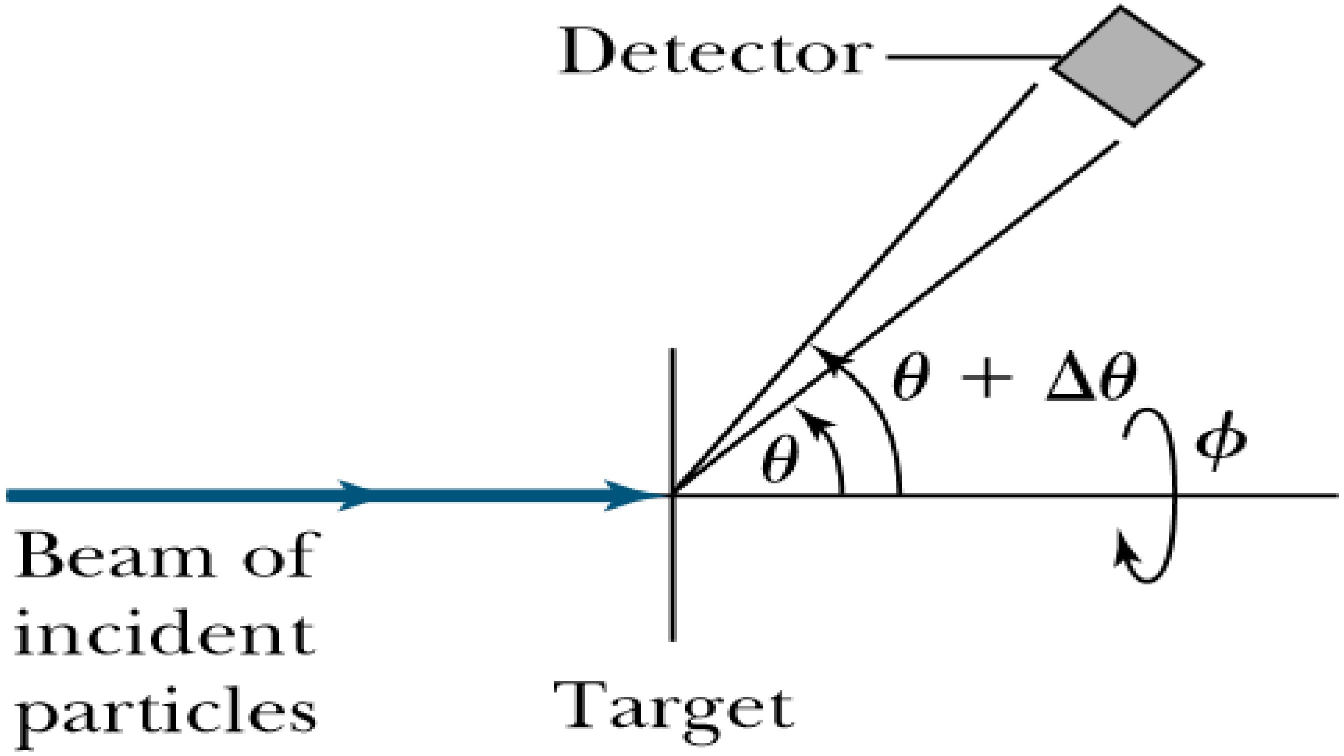
The probability of the particle being scattered is equal to the total target area exposed for all the nuclei divided by the total target area A. If σ is the cross section for each nucleus, then $ntA\sigma$ is the total area exposed by the target nuclei, and the fraction of incident particles scattered by an angle of θ or greater is

$$f = \frac{\text{target area exposed by scatterers}}{\text{total target area}} = \frac{ntA\sigma}{A}$$

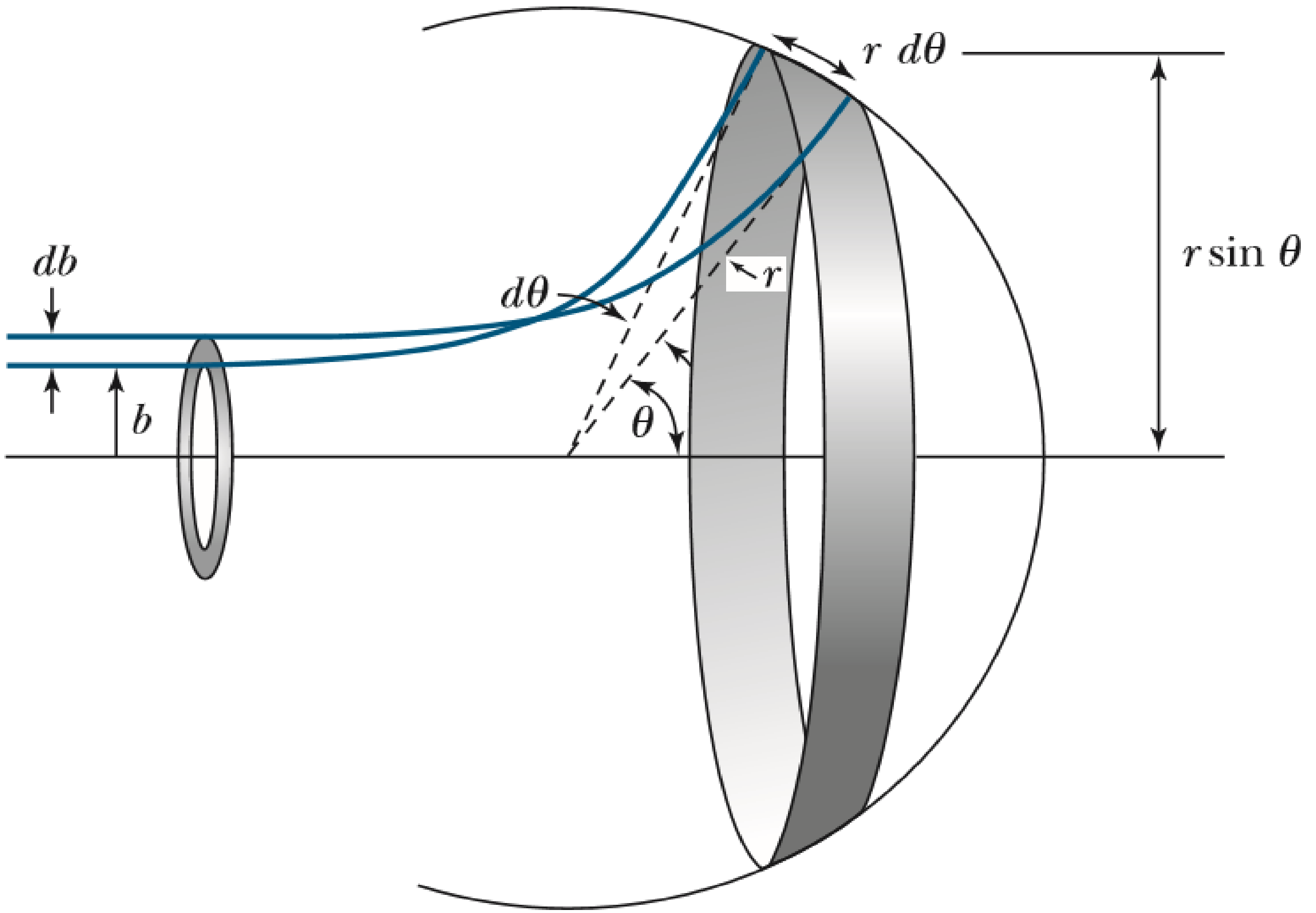
$$= nt\sigma = nt\pi b^2$$

$$f = \pi nt \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K} \right)^2 \cot^2 \frac{\theta}{2}$$

- In a typical experiment, however, a detector is positioned over a range of angles from θ to θ+Δθ , as shown in Figure.



- Thus we need to find **the number of particles scattered between** θ and θ+Δθ that corresponds to incident particles with impact parameters between b and b+db as displayed in Figure.



- The **fraction** of the incident particles scattered between θ+Δθ is df.
- The derivative of f

$$df = -\pi nt\left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\right)^2 \cot\frac{\theta}{2}\csc^2\frac{\theta}{2}\, d\theta$$

If the total number of incident particles is N_i, the number of particles scattered into the ring of angular width $d\theta$ is $N_i|df|$. The area dA into which the particles scatter is $(r\, d\theta)(2\pi r \sin\theta) = 2\pi r^2 \sin\theta\, d\theta$. Therefore, the number of particles scattered per unit area, $N(\theta)$, into the ring at scattering angle θ is

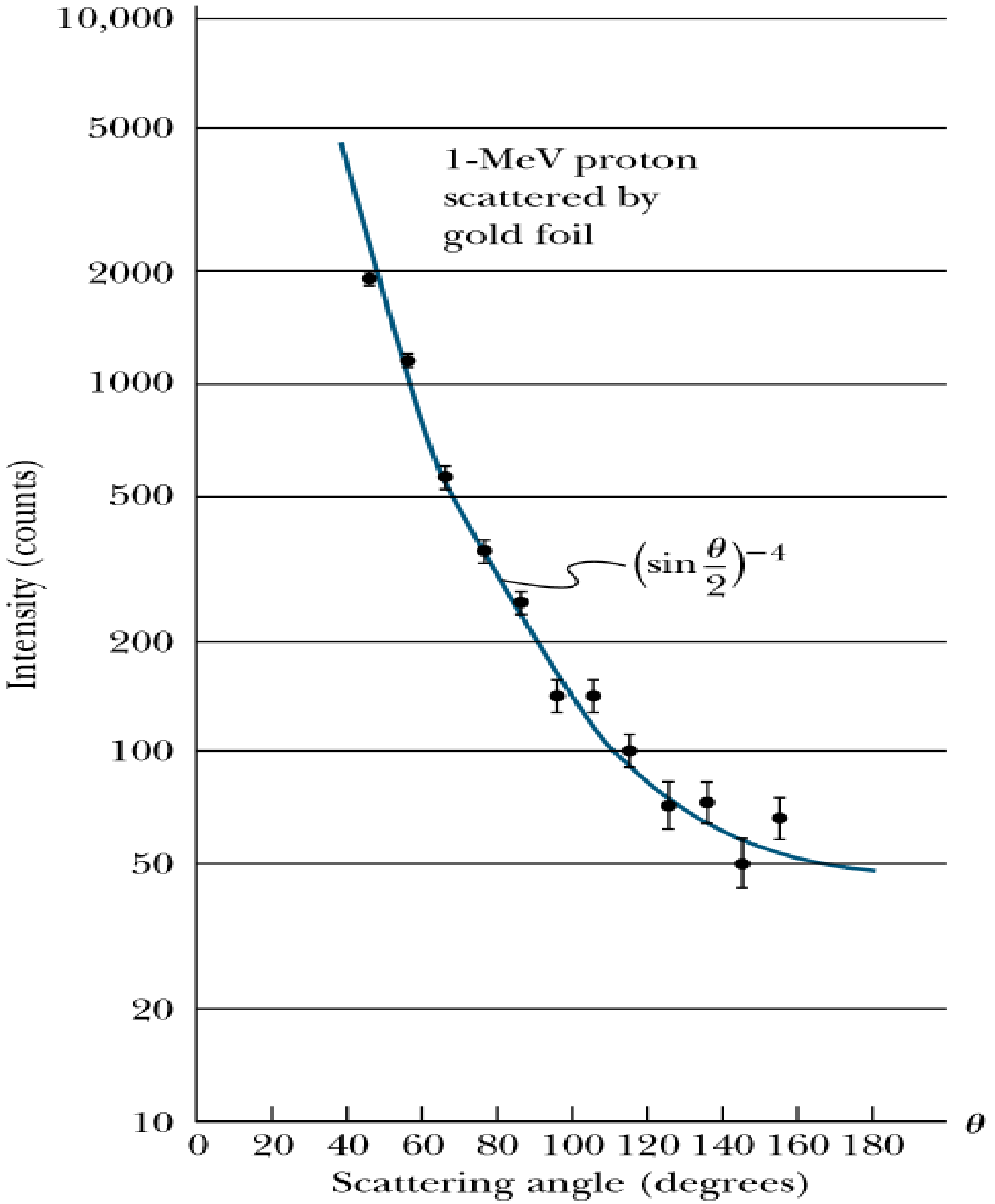
$$N(\theta) = \frac{N_i|df|}{dA} = \frac{N_i \pi nt\left(\dfrac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\right)^2}{2\pi r^2 \sin\theta\, d\theta}\cot\frac{\theta}{2}\csc^2\frac{\theta}{2}\, d\theta$$

$$N(\theta) = \frac{N_i nt}{16}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{Z_1^2 Z_2^2}{r^2 K^2 \sin^4(\theta/2)}$$

Rutherford scattering equation

The important points of Rutherford scattering equation are the following :

1. The scattering is proportional to the square of the atomic number of both the incident particle (Z_1) and the target scatterer (Z_2).
2. The number of scattered particles is inversely proportional to the square of the kinetic energy K of the incident particle.
3. The scattering is inversely proportional to the fourth power of $\sin(\theta/2)$, where θ is the scattering angle.
4. The scattering is proportional to the target thickness for thin targets.



- These specific predictions by Rutherford in 1911 were confirmed experimentally by Geiger and Marsden in 1913.

- The angular dependence is particularly characteristic and can be verified in a well-equipped undergraduate physics laboratory, as we see from some actual data shown in Figure.

The Classical Atomic Model

- ✓After Rutherford presented his calculations of charged-particle scattering in 1911 and the experimental verification by his group in 1913, it was generally conceded that the atom consisted of a small, massive, positively charged “nucleus” surrounded by moving electrons.
- ✓Thomson’s plum-pudding model was definitively excluded by the data.
- ✓Actually, Thomson had previously considered a planetary model resembling the solar system (in which the planets move in elliptical orbits about the sun) but rejected it because, although both gravitational and Coulomb forces vary inversely with the square of the distance, the planets attract one another while orbiting around the sun, whereas the electrons would repel one another.
- ✓Thomson considered this to be a fatal flaw from his knowledge of planetary theory.

In order to examine the failure of the planetary model, let us examine the simplest atom, hydrogen. We will assume circular electron orbits for simplicity rather than the more general elliptical ones. The force of attraction on the electron due to the nucleus (charge $= +e$) is

$$\vec{F}_e = \frac{-1}{4\pi\epsilon_0}\frac{e^2}{r^2}\,\hat{e}_r$$

where the negative sign indicates the force is attractive and $\hat{e}_r$ is a unit vector in the direction from the nucleus to the electron. This electrostatic force provides the centripetal force needed for the electron to move in a circular orbit at constant speed. Its radial acceleration is

$$a_r = \frac{v^2}{r}$$

where v is the tangential velocity of the electron. Newton's second law now gives

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

and

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}}$$

where we are using m without a subscript to be the electron's mass. When it is not clear what particle m refers to, we write the electron mass as m_e.

The kinetic energy of the system is due to the electron, $K = mv^2/2$. The nucleus is so massive compared with the electron ($m_{\text{proton}} = 1836m$) that the nucleus may be considered to be at rest. The potential energy V is simply $-e^2/4\pi\epsilon_0 r$, so the total mechanical energy is

$$E = K + V = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

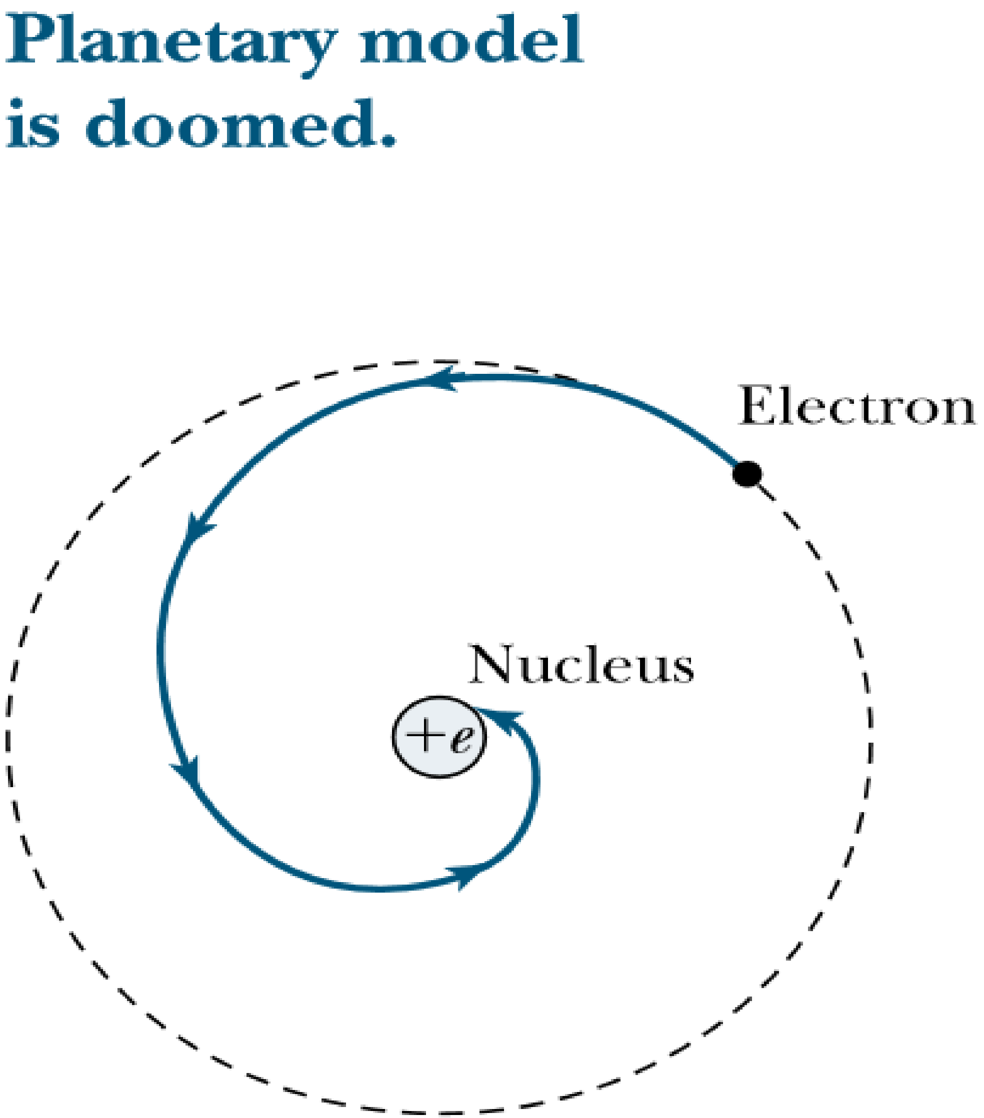
Replace the v derived in the equation.

$$E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = \frac{-e^2}{8\pi\epsilon_0 r}$$

The total energy is negative, indicating a bound system.

- Thus far, the classical atomic model seems plausible.
- **The problem arises when we consider that the electron is accelerating due to its circular motion about the nucleus.**
- We know from classical electromagnetic theory that an accelerated electric charge continuously radiates energy in the form of electromagnetic radiation.
- If the electron is radiating energy, then the total energy E of the system must decrease continuously.
- In order for this to happen, the radius r must decrease.
- The electron will continuously radiate energy as the electron orbit becomes smaller and smaller until the electron crashes into the nucleus!

- This process, displayed in Figure, would occur in about 10^{-9} s.



- Thus the classical theories of Newton and Maxwell, which had served Rutherford so well in his analysis of a-particle scattering and had thereby enabled him to discover the nucleus, also led to the failure of the planetary model of the atom.
- **Physics had reached a decisive turning point like that encountered in 1900 with Planck's revolutionary hypothesis of the quantum behavior of radiation.**
- In the early 1910s, however, the answer would not be long in coming, as we shall see in the next section.

The Bohr Model of the Hydrogen Atom

- **Bohr**, believed that a fundamental length about the size of an atom (10^{-10} m) was needed for an atomic model.
- This fundamental length might somehow be connected to **Planck's** new constant h.
- The pieces finally came together when **Bohr** learned of new precise measurements of the hydrogen spectrum and of the empirical formulas describing them.
- He set out to find a fundamental basis from which to derive the Balmer formula, the Rydberg equation and Ritz's combination principles.

- Bohr was well acquainted with Planck's work on the quantum nature of radiation.
- Like Einstein, Bohr believed that quantum principles should govern more phenomena than just the blackbody spectrum.
- He was impressed by Einstein's application of the quantum theory to the photoelectric effect and to the specific heat of solids and wondered how the quantum theory might affect atomic structure.
- Bohr assumed that electrons moved around a massive, positively charged nucleus.
- Bohr assume for simplicity that the electron orbits are circular rather than elliptical and that the nuclear mass is so much greater than the electron's mass that it may be taken to be infinite.
- The electron has charge -e and mass m and revolves around a nucleus of charge +e in a circle of radius α.
- The size of the nucleus is small compared with the atomic radius α.

Bohr’s general assumptions

- Bohr’s model may best be summarized by the following “general assumptions” of his 1915 paper:

A. Certain “stationary states” exist in atoms, which differ from the classi-cal stable states in that the orbiting electrons do not continuously radiate electromagnetic energy. The stationary states are states of definite total energy.

B. The emission or absorption of electromagnetic radiation can occur only in conjunction with a transition between two stationary states. The frequency of the emitted or absorbed radiation is proportional to the difference in energy of the two stationary states (1 and 2):

$$E = E_1 - E_2 = hf$$

where h is Planck’s constant.

C. The dynamical equilibrium of the system in the stationary states is governed by classical laws of physics, but these laws do not apply to transitions between stationary states.

D. The mean value K of the kinetic energy of the electron-nucleus system is given by $K = nhf_{orb}/2$, where f_{orb} is the frequency of rotation. For a circular orbit, Bohr pointed out that this assumption is equivalent to the angular momentum of the system in a stationary state being an integral multiple of h/2π. (ħ=h/2π, pronounced “ħ bar.”)

- These four assumptions were all that Bohr needed to derive the Rydberg equation.

- Bohr believed that atoms exist and do not continuously radiate energy: atoms were stable
- It also seemed that the classical laws of physics could not explain the observed behavior of the atom.
- By the quantization of angular momentum aspect, a particularly simple derivation of the Rydberg equation is possible.
- Let us now proceed to derive the Rydberg equation using Bohr’s assumptions.

- The total energy (potential plus kinetic) of a hydrogen atom was derived previously.

$$E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = \frac{-e^2}{8\pi\epsilon_0 r}$$

- For circular motion, the magnitude of the angular momentum L of the electron is

$$L = \left|\vec{r} \times \vec{p}\right| = mvr$$

Assumption D states this should equal $n\hbar$:

$$L = mvr = n\hbar$$

where n is an integer called the **principal quantum number**. We solve the previous equation for the velocity and obtain

Principal quantum number

$$v = \frac{n\hbar}{mr}$$

We set equal the two velocity equations and ve receive this formula:

$$v^2 = \frac{e^2}{4\pi\epsilon_0 mr} = \frac{n^2\hbar^2}{m^2r^2}$$

We see that only certain values of r are allowed.

$$r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{me^2} \equiv n^2 a_0$$

where the **Bohr radius** a_0 is given by

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2}$$

$$= \frac{(1.055 \times 10^{-34}\,\mathrm{J\cdot s})^2}{\left(8.99 \times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(9.11 \times 10^{-31}\,\mathrm{kg})(1.6 \times 10^{-19}\,\mathrm{C})^2}$$

$$= 0.53 \times 10^{-10}\,\mathrm{m}$$

Bohr radius

- Notice that the smallest diameter of the hydrogen atom is $2r_1 = 2a_0 \approx 10^{-10}\, m$, the suspected (now known) size of the hydrogen atom!
- Fundamental length a_0 , is determined for the value n =1.
- The atomic radius is now quantized.
- The quantization of various physical values arises because of the principal quantum number n.
- The value n =1 gives the radius of the hydrogen atom in its lowest energy state (called the “ground” state).
- The values of n > 1 determine other possible radii where the hydrogen atom is in an “excited” state.

The energies of the stationary states can now be determined

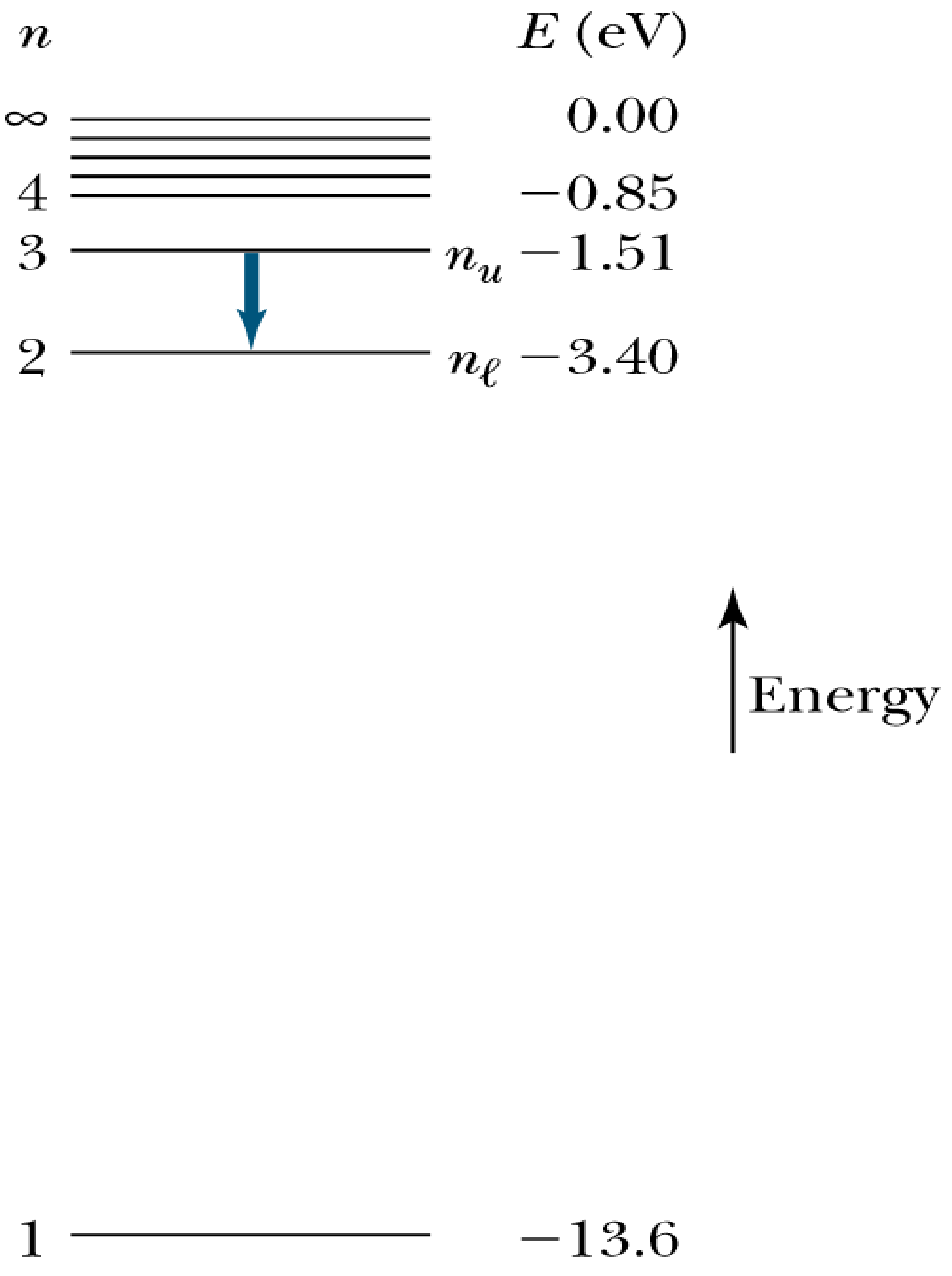
$$E_n = -\frac{e^2}{8\pi\epsilon_0 r_n} = -\frac{e^2}{8\pi\epsilon_0 a_0 n^2} \equiv -\frac{E_0}{n^2}$$

Quantized energy states

The lowest energy state ($n = 1$) is $E_1 = -E_0$ where

$$E_0 = \frac{e^2}{8\pi\epsilon_0 a_0} = \frac{e^2}{(8\pi\epsilon_0)}\frac{me^2}{4\pi\epsilon_0\hbar^2} = \frac{me^4}{2\hbar^2(4\pi\epsilon_0)^2} = 13.6\text{ eV}$$

- This is the experimentally measured ionization energy of the hydrogen atom.



- Bohr's Assumptions C and D imply that the atom can exist only in "stationary states" with definite, quantized $energies\, E_n$, displayed in the **energy-level diagram** of Figure.

- Emission of a quantum of light occurs when the atom is in an excited state (quantum number $n = n_u$) and decays to a lower energy state ($n = nl$).

- A transition between two energy levels is schematically illustrated in Figure.
- According to Assumption B we have

$$hf = E_u - E_\ell$$

where f is the frequency of the emitted light quantum (photon). Because $\lambda f = c$, we have

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E_u - E_\ell}{hc}$$

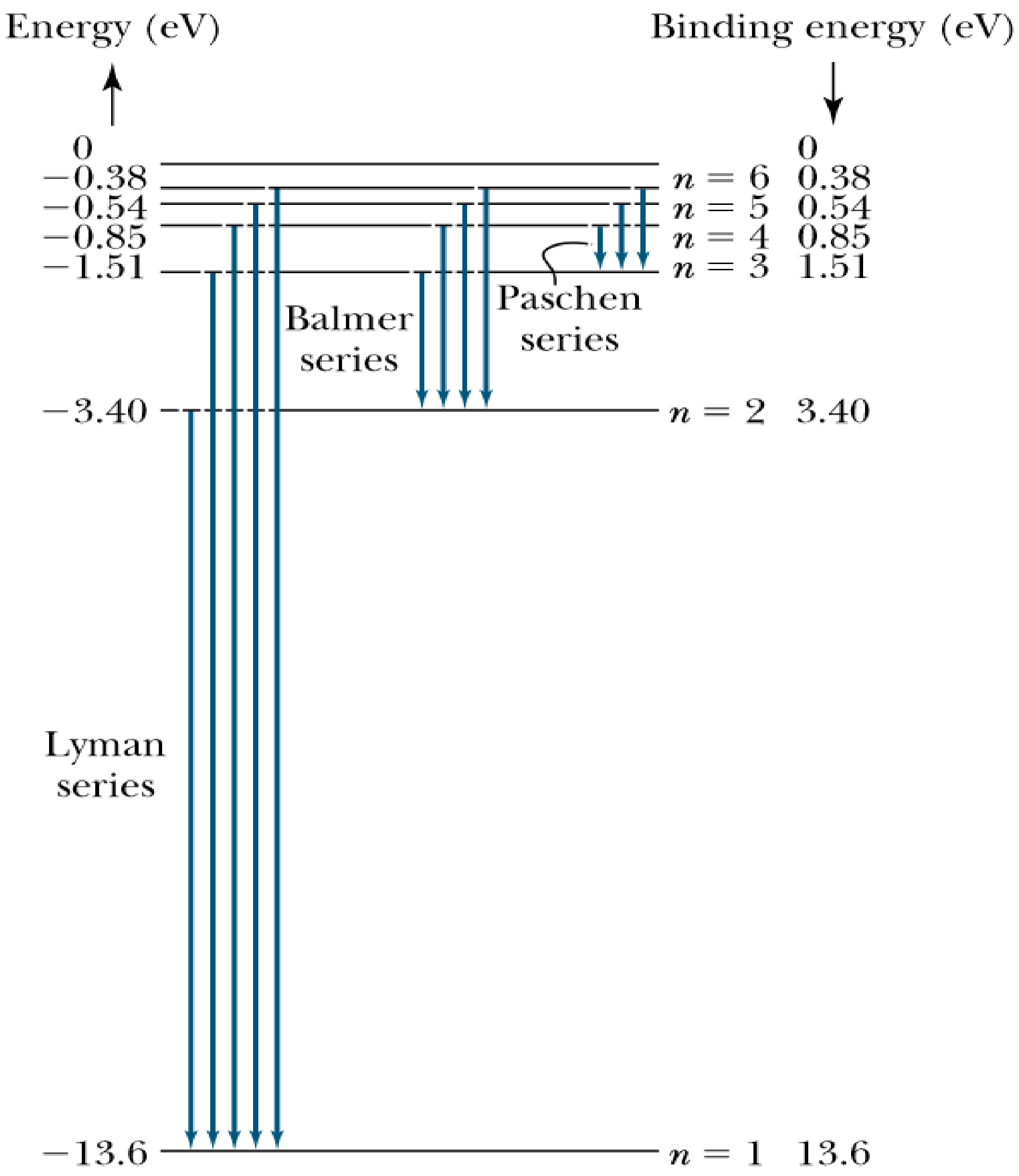
$$= \frac{-E_0}{hc}\left(\frac{1}{n_u{}^2} - \frac{1}{n_\ell{}^2}\right) = \frac{E_0}{hc}\left(\frac{1}{n_\ell{}^2} - \frac{1}{n_u{}^2}\right)$$

where

$$\frac{E_0}{hc} = \frac{me^4}{4\pi c\hbar^3(4\pi\epsilon_0)^2} \equiv R_\infty$$

This constant R_∞ is called the **Rydberg constant** (for an infinite nuclear mass).

$$\frac{1}{\lambda} = R_\infty\left(\frac{1}{n_\ell{}^2} - \frac{1}{n_u{}^2}\right)$$ which is similar to the Rydberg equation.



- The value of $R_\infty = 1.097373\ x\ 10^7\ m^{-1}$ calculated from Equation agrees well with the experimental and we will obtain an even more accurate result in the next section.
- Bohr's model predicts the frequencies (and wavelengths) of all possible transitions in atomic hydrogen.
- Several of the series are shown in Figure.
- The Lyman series represents transitions to the lowest state with n_l =1; the Balmer series results from downward transitions to the stationary state n_l = 2; and the Paschen series represents transitions to n_l = 3.

- Bohr had successfully accounted for the known spectral lines of hydrogen.
- The frequencies of the photons in the emission spectrum of an element are directly proportional to the differences in energy of the stationary states.
- When we pass white light (composed of all visible photon frequencies) through atomic hydrogen gas, we find that certain frequencies are absent.
- This pattern of dark lines is called an **absorption spectrum**.
- The missing frequencies are precisely the ones observed in the corresponding **emission spectrum**.

- In absorption, certain photons of light are absorbed, giving up energy to the atom and enabling the electron to move from a lower (/) to a higher (u) stationary state.
- The atom will remain in the excited state for only a short time (on the order of 10^{-10} s) before emitting a photon and returning to a lower stationary state.
- Thus, at ordinary temperatures practically all hydrogen atoms exist in the lowest possible energy state, n =1, and only the absorption spectral lines of the Lyman series are normally observed.
- However, these lines are not in the visible region.
- The sun produces electromagnetic radiation over a wide range of wavelengths, including the visible region.
- When sunlight passes through the sun’s outer atmosphere, its hydrogen atoms absorb the wavelengths of the Balmer series (visible region), and the absorption spectrum has dark lines at the known wavelengths of the Balmer series.

- We can determine the electron's velocity in the Bohr model

$$v_n = \frac{n\hbar}{mr_n} = \frac{n\hbar}{mn^2 a_0} = \frac{1}{n}\frac{\hbar}{ma_0}$$

or

$$v_n = \frac{1}{n}\frac{e^2}{4\pi\epsilon_0\hbar}$$

The value of v_1 is $\hbar/ma_0 = 2.2 \times 10^6$ m/s, which is less than 1% of the speed of light. We define the dimensionless quantity ratio of v_1 to c as

$$\alpha \equiv \frac{v_1}{c} = \frac{\hbar}{ma_0 c} = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

Fine structure constant

This ratio is called the **fine structure constant**. It appears often in atomic structure calculations.

- Bohr’s atomic model of quantized energy levels represented a significant step forward in understanding the structure of the atom.
- Although it had many successes, we know now that, in principle, it is wrong.
- We will discuss some of its successes and failures in the next section and discuss the correct quantum theory.
- Nevertheless, Bohr’s atomic model is useful in our first attempt in understanding the structure of the atom.